Exact solution of the optical Bloch equation for the Demkov model

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Coherent excitation influenced by dephasing processes represents an important topic in quantum mechanics [1, 2]. Applications of such models are numerous ranging from coherent atomic excitation and quantum information to chemical physics and solid-state physics. Although significant efforts have been devoted to studying the Bloch equations describing specific two-state models, almost all results are related to some asymptotic regimes such as weak dephasing, strong coupling or other limits [4]. There are very few exact solutions to the Bloch equation. The complexity of this problem is due to the difficulty of deriving an exact solution to third order linear differential equations. For resonant coherent excitation of a two-state system in the presence of dephasing, a solution can be found in [5].

We model the time evolution of the density operator for the two-level system in the presence of dephasing by the following master equation

$$\frac{d}{dt}\rho(t) = -\mathrm{i}[H,\rho] + \frac{\Gamma}{2}(\sigma_z\rho\sigma_z - \rho), \tag{1}$$

where $\Gamma = 1/t_{\text{dec}}$ is the constant decay rate and t_{dec} is the decoherence time. The Bloch vector description of the dissipative dynamics yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} u(t) \\ v(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} -\Gamma & -\Delta & 0 \\ \Delta & -\Gamma & -\Omega(t) \\ 0 & \Omega(t) & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ v(t) \\ w(t) \end{bmatrix}, \tag{2}$$

where the components of the Bloch vector $\vec{B}(t) = [u(t), v(t), w(t)]^T$ are expressed via density matrix elements $\rho_{ij} = \langle i|\rho|j\rangle$, as follows

$$u = 2\Re\rho_{\uparrow\downarrow},\ v = 2\Im\rho_{\uparrow\downarrow},\ w = \rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow}. \tag{3}$$

We obtain a third order differential equation for the population inversion w, which reads,

$$\dddot{w} + 2(T^{-1}+\Gamma)\ddot{w} + \left[(T^{-1}+\Gamma)^2 + \Delta^2 + \Omega_0^2 e^{-2t/T}\right]\dot{w} + \Omega_0^2(-T^{-1}+\Gamma)e^{-2t/T}w = 0. \tag{4}$$

Next we transform (4) to a third order hypergeometric equation, whose solution we investigate further and derive asymptotic for.

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