Multi-Photon Scattering Theory and Generalized Master Equation

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Exploring devices inducing the strong photon-photon interaction has inspired a lot of interest and extensive studies. These devices provide us an platform to investigate the generation and transport of the non-classical light, e.g., the single- photon source and switch, which are the crucial ingredients in quantum optics and quantum information processing. The manipulation of non-classical lights requires the devices with the strong nonlinearity and quantum interference effect, which induce the exotic quantum statistics behaviors of photons emitted from the system, e.g., the anti-bunching behaviors in the generation of the single- photon and the single photon pairs. Such devices have been designed in different systems, e.g., the cavity quantum electrodynamics (QED) systems, the Nitrogen-vacancy (NV) center systems, and the circuit QED systems. In the many-particle level, the strong interaction between the nonlinear devices and photons results in the novel many-body states, which can be studied by the quantum spin model for the nonlinear systems. The investigations of the steady state behaviors and the photon transmission properties in the quantum spin model can reveal the rich quantum phases and photon statistics in the dissipative nonlinear devices.

To understand the mechanism of manipulating non-classical lights, one could study the transmission spectra and the quantum statistics characterized by the second order correlation function of photons emitted from the nonlinear system under the weak driving light. This is the typical problem in quantum optics, where the input-output theory relates the correlation functions of the emitted photons to those of the system fields in the steady state obtained by the master equation approach. However, there are some disadvantages in this conventional method. First, conceptually, in the master equation approach, the Born-Markov approximation is used. Even though it is a very good approximation for most of the models in quantum optics, can one develop an exact approach to justify the application of the Born-Markov approximation in the system with the strong nonlinearity and quantum interferences? Secondly, in practice, the master equation is solved in the enlarged super-space, which is only able to give the numerical results in general. Thus, is there a simple method, which can provide us some analytical results to understand the mechanism better?

It turns out that the few-photon scattering theory is the proper way to solve the above problems. Intuitively, since the driving intensity is weak, the driven system can be considered as the scattering of few incident photons by the system without any driving field. The few-photon scattering by the quantum emitters in the one-dimensional (1D) waveguide is extensively studied by different elegant methods, e.g., the Bethe-ansatz method, the input-output formalism, and the Lehmann-Symanzik-Zimmerman (LSZ) reduction formalism, which lead to the photonic S-matrix determining the quantum properties of the out-going photons.

Although the scattering theory is very successful in the study of the photon transmission to the quantum devices, to our best knowledge, there are still some interesting questions, which are not explored before. The comprehensive understanding of transient behaviors is needed for the photon detection processes, e.g., the single-photon edge sensor in the superconducting systems. In order to describe the transient processes, one could use the master equation approach to study the time evolution of the system reduced density matrix. However, for the scattering problem, where the initial state of the bath contains few photon excitations, it turns out that the conventional master equation breaks down. On the other hand, a natural question arises, namely, what is the exact relation between the weak driven system and the few-photon transport in the system.

In order to address the above problems, in this paper, we develop the scattering theory to study the few-photon scattering by the quantum emitters in the 1D waveguide by two approaches, i.e., the displacement transformation method and the path integral formalism. Here, the time evolution of the total system and the transition amplitudes between the arbitrary initial and final states are obtained analytically. By the different boundary conditions for the initial and final states, the scattering amplitude determined by the S-matrix, the spontaneous and stimulated emissions, and all transient behaviors including the response of the system to the few incident photons can be studied by the developed scattering theory. By the same methods, we derive the generalized master equation for the system reduced density matrix to describe the time evolution of the quantum emitters for the bath containing the few incident photons.

The photon scattering by an array of Rydberg-electromagnetic induce transparency (EIT) atoms is studied by the developed scattering theory. Here, the interplay between the Rydberg interaction and the EIT effect results in the rich quantum statistics of two outgoing photons. The co- and counter- propagations of two polaritons in the atom array for two incident photon wavepacket are studied, which show the anti-bunched co-propagating dark polaritons and the collision behaviors of the counter-propagating polaritons in the transient processes.