A quantum private states of a dimension \(d\) (so called pdits) is composed from a set \(d \otimes d A, B\) part called a "key", and \(A', B'\) called "shield", shared between Alice (subsystems \(A, A')\) and Bob (subsystems \(B, B')\) in such a way that the local von Neumann measurements on the key part in a particular basis will make its results completely statistically uncorrelated from the results of any measurement of an eavesdropper Eve on her subsystem \(E\), which is a part of the purification \(\psi_{ABA'B'E}\) of the pdit state \(\rho_{ABA'B'}.\) Pdits (especially pdits) are of great importance in quantum cryptography and have been studied extensively for some time. We present a construction of quantum states in dimension \(d\) that is nearly as far from separable ones as possible.

Let us consider the following state \([1]\):

\[
\rho_{ABA'B'} = \sum_{i=0}^{d} \sum_{j=0}^{d-1} \omega_i \in \mathcal{B}\left(\mathcal{H}_{d_i} \otimes \mathcal{H}_{d_i} \otimes \mathcal{H}_{d_k} \otimes \mathcal{H}_{d_k}\right),
\]

where \(\mathcal{B}(\mathcal{H})\) is the algebra of all bounded linear operators on Hilbert space \(\mathcal{H}\), and \(d = \frac{d}{2}d_k(d_k - 1)\) and by \(d_k\) we denote the dimension of the key part acting on \(A'B'\). Now we describe each of the components from Eq. \([1]\). First of all, we define the term \(\omega_0\) as:

\[
\omega_0 = \sum_{i,j=0}^{d-1} |i\rangle \langle i| \otimes |j\rangle \langle j| \otimes a_{ij}^{(0,0)},
\]

where every \(a_{ij}^{(0,0)} \in \mathcal{B}\left(\mathcal{H}_{d_i} \otimes \mathcal{H}_{d_k}\right)\). From now on, every matrix of the form Eq. \([2]\) we will call matrix in the maximally entangled form. The rest of elements \(\omega_l\), for \(1 \leq l \leq 1 = \frac{1}{2}d_k(d_k - 1)\) from \([1]\) are given by the following formula:

\[
\omega_l = |i\rangle \langle i| \otimes |j\rangle \langle j| \otimes a_{ij}^{(i,j)} + |i\rangle \langle j| \otimes |j\rangle \langle i| \otimes a_{ij}^{(i,j)},
\]

where \(i, j = 1, \ldots, d_k - 1\) and \(i < j\). In the above we also implicitly assume bijective function between indices \(l\) and \(i, j\).

We formulate and prove the following lemmas and theorem concerning our construction of private states \([1]\)

**Lemma 1.** Let us assume that we are given with \(\rho_{ABA'B'}\) as in Eq. \([1]\) and the pdit \(\gamma_0\) in its maximally entangled form, then the following statement holds:

\[
\|\rho_{ABA'B'} - \gamma_0\|_1 = 2q.
\]

where \(0 \leq q \leq 1\).

**Lemma 2.** Let us consider the class of states given by

\[
\rho_{ABA'B'} = p\gamma_0 + \frac{q}{d} \sum_{i=1}^{d} \gamma_i,
\]

where \(q = 1 - p\) and \(d = \frac{1}{2}d_k(d_k - 1)\) and states \(\gamma_0, \gamma_i\) are given by Eq. \([2]\). Then the trace distance from the set of private dits in maximally entangled form is equal to

\[
\frac{1}{2} \|\rho_{ABA'B'} - \gamma_0\|_1 = \frac{1}{1 + \frac{d}{d_k - 1}},
\]

where \(d_k\) is the dimension of the key part and \(d_k\) - the dimension of the shield part.

**Theorem 4.** For an arbitrary \(\epsilon > 0\) there exists a PPT state \(\rho\) acting on the Hilbert space \(\mathbb{C}^d \otimes \mathbb{C}^d\) with \(d \leq \frac{4}{\epsilon^2}\) such that:

\[
dist(\rho, \mathcal{SEP}) \geq 2 - \epsilon,
\]

where \(c\) is constant.

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