Inequalities of Nonlinearly Tansformed X-states

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Our talk is devoted to nonlinear transformations [1, 2] of density matrices. The aim ot the talk is to generalize the nonlinear map (quantum channel) considered in [3] and discuss the properties of states obtained in result of that channel action.

The previously considered nonlinear map was

$$\Phi_n(\rho) = \frac{\rho^n}{\mathrm{Tr}\rho^n}, \qquad (1)$$

where ρ is the density matrix of the initial state. The quantum channel is determined by the integer parameter n. We generlize this quantum channel by introducing the new one depending on arbitrary real number h.

We consider in our work the states described by 4×4 density matrix ρ_X of specific type, called X-state [4]. The density matrix ρ_X is defined by the formula

$$\rho_X = \begin{pmatrix} a & 0 & 0 & f \\ 0 & b & e & 0 \\ 0 & e^* & c & 0 \\ f^* & 0 & c & d \end{pmatrix}, \qquad (2)$$

where a, b, c, d, e and f are parameters. By varying these numbers one can obtain the density matrices of states with different entanglement properties. The matrix of such type can descibe two-qubit systems or the qudit with $j = 3/2$. For example, in case of two spins (each with possible states $\uparrow$ and $\downarrow$) the indeces of matrix could be choosen to correspond to states $\uparrow\uparrow$, $\uparrow\downarrow$, $\downarrow\uparrow$, $\downarrow\downarrow$. For qudit $j = 3/2$ we can choose the rows and colomns of matrix ρ_X to correspond to projection of angular momentum. Thus, the matrices of considered type provide possibility to study the action of nonlinear channel on wide variety of states.

In our work we discuss the entanglement of the states produced by considered quantum channels. We use Peres-Horodecki criterion [5, 6] to testify if the state entangled or not. The negativity and concurrence, which are numerical characteristics of entanglement, of new states are discussed. The channels defined by different real numbers h can produce states with different entanglement properties in result of action on the same initial state. In particular, the considered nonlinear maps can produce entangled state in result of action on separable state.

It is interesting to investigate the action of nonlinear channel on entropic and information properties of states.

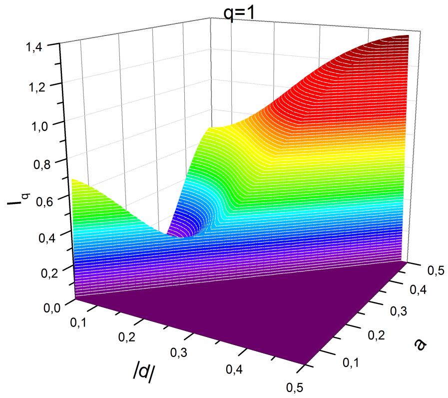

FIG. 1. Tsallis information for transformed X-state $\Phi_2(\rho_X)$ with parameters $a = d$, $b = c$, $e = 0.01$ for $q = 1$.

We consider the Tsallis entropy (also known as q-entropy) defined by real parameter q. The Neumann entropy is particular case of Tsallis entropy at the limit $q \to 1$. Thus, we study more general definition of entropy than Neumann entropy and consider the Neumann entropy as the particular case. The Tsallis information is defined analogously to Neumann information. We disscuss the information for different values of parameters h of nonlinear maps and for different values of parameter q of q-entropy.

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