IFE states under time dependent Hamiltonians

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An interaction-free evolution (IFE) of a quantum system is an evolution which is not influenced by a certain part of the Hamiltonian which is addressed as the interaction term [1]. In other words, the dynamics generated by the 'unperturbed' Hamiltonian H_0 is essentially the same as the evolution generated by the total Hamiltonian which is the sum of H_0 and the interaction term H_I: $H = H_0 + H_\mathrm{I}$. This notion, which has been introduced in Ref. [1], is somehow related to the concept of decoherence-free subspaces (DFS) [2–4]. In spite of such connection, it should be stressed that the two concepts are still different in many aspects. Generally speaking, the notion of IFE can be relevant to composite systems with different dimensions (like a small system and its environment) or with similar dimensions (for example two interacting qubits), but it can even concern different degrees of freedom of the same particle (for example atomic and vibrational degrees of freedom of a trapped ion). One can even talk about IFE states in connection with the action of a classical field on a quantum system, for example a spin under the action of a magnetic field.

Subradiance [5], in its original formulation, is surely a very famous phenomenon which can be thought of as an IFE involving a matter system (several atoms) and the vacuum electromagnetic field.

Here we study the non trivial extension of IFE states which applies to those cases wherein the system is governed by a time-dependent Hamiltonian. The interest in such a kind of problem is related to several aspects. On the one hand, generally speaking the resolution of dynamical problems with time-dependent Hamiltonians is a tough job due to the highly nontrivial structure of the corresponding solution

$$U(t) = \mathcal{T} \exp\left(-\mathrm{i} \int_0^t H(\tau) \mathrm{d}\tau\right), \qquad (1)$$

where $\mathcal{T}$ denotes the chronological product. In general Eq. (1) is untractable and, except for some lucky cases [6–8], it requires special assumptions, such as for example the adiabatic one [9], or suitable approximations, like in the perturbative treatment [10, 11]. Therefore, even the partial resolution of a class of time-dependent problems in the presence of time-dependent Hamiltonians is of interest itself. Formula (1) simplifies if $H(t)$ defines a commutative family, i.e. $[H(t), H(t')] = 0$ for arbitrary t and t'. In this case the chronological product drops out and the entire evolution is controlled by the integral $\int_0^t H(\tau)\mathrm{d}\tau$.

On the other hand, there could be important applications in the field of quantum control and in particular in the field of suppression of decoherence effects. Indeed, our analysis, could pave the way to extensions of the concepts of subradiance and decoherence-free subspaces in the presence of time-dependent Hamiltonian of the system and even in the presence of time-dependent interaction between the system and its environment.

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