Study of the elastic enhancement factor and nearest neighbor spacing distribution for partially chaotic and chaotic systems

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We will present the results of experimental studies of the elastic enhancement factor W and the nearest neighbors spacing distribution P(s) for microwave rectangular and rough cavities simulating two-dimensional quantum billiards in a transient region between regular and chaotic dynamics [1] and in a pure chaotic dynamics region, respectively. The analogy between microwave cavities and quantum billiards is based upon the equivalency of the Helmholtz equation describing 2D microwave cavities and the Schrödinger equation describing the quantum systems [2]. It holds for the excitation frequency below $\nu_{max} = c/2d$, where c is the speed of light in the vacuum and d is the height of the cavity, when only the transverse magnetic TM_0 mode can be excited inside the cavity.

The elastic enhancement factor W is the ratio of variances of diagonal elements of the two-port scattering matrix S to off-diagonal elements of this matrix. W is defined by the relationship [3, 4]:

$$W_\beta = \frac{\sqrt{\mathrm{var}(S_{aa})\mathrm{var}(S_{bb})}}{\mathrm{var}(S_{ab})},$$

where $var(S_{ab}) \equiv \langle |S_{ab}|^2 \rangle - |\langle S_{ab} \rangle|^2$ is the variance of the scattering matrix element Sab of the two-port scattering matrix, and $\beta = 1, 2$ for systems with preserved and broken time reversal symmetry, respectively.

The nearest neighbor spacing distribution for the regular system is given by the Poisson distribution, whereas for the chaotic system with preserved time reversal symmetry it is Wigner distribution $p_1(s) = \frac{\pi}{2} s e^{-\frac{\pi}{4}s^2}$

In order to measure spectra (scattering matrices) of rectangular and rough cavities we used a vector network analyzer Agilent E8364B, which was connected to antennas introduced inside a cavity through the flexible microwave cables HP 85133-616 and HP 85133-617. The measurements were done in the frequency range 16 - 18.5 GHz for a moderate absorption $\gamma = 5.2$-7.4

The results obtained for the elastic enhancement factor W and the nearest neighbor distribution for the rectangular cavity are not in the agreement with the theoretical prediction for the integrable systems. On the other hand, they are significantly different than the ones obtained for the microwave rough cavity, simulating chaotic billiard, which in case of the enhancement factor turned out to lie between the results predicted by RMT and the ones predicted within a recently introduced model of the two-channel coupling [5, 6]. The results for the rectangular cavity may be explained by taking into account a scattering on the antennas, which causes that the system becomes partially chaotic. We found that in our experiment a transient parameter describing the departure from the integrability equals k = 2.8. Our experimental results suggest that the enhancement factor can be used as a measure of internal chaos that can be especially useful for systems with significant absorption or openness.

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