Squeezed-entangled states in optical quantum metrology

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Quantum metrology exploits quantum correlations to make precise measurements with limited particle numbers. An indispensable tool in the field of metrology is the interferometer; here we utilise the specific Mach-Zehnder interferometer (MZI) which allows us to measure phase differences with extreme precision. Sending classical states of light into the MZI results in a precision that scales with the number of photons as $1/\sqrt{N}$, this is the well known shot-noise limit (SNL). However, quantum mechanics permits correlations between photons which can be exploited to significantly enhance the precision surpassing the SNL. An example of such a highly correlated state of the same number of N photons is given by the NOON state $|\psi_{\text{NOON}}\rangle = \frac{1}{\sqrt{2}}(|N,0\rangle_{a,b} + |0,N\rangle_{a,b})$ which yields a precision scaling of $1/N$ known as the Heisenberg limit [1].

The correlations associated with the NOON state are present between the spatial modes of the interferometer in the form of entanglement. An alternative to these inter-mode correlations are intra-mode correlations such as those given by the squeezing operation which is mathematically described by $\hat{S}(z) = \exp\left(\frac{1}{2}(z^* a^2 + z a^{\dagger 2})\right)$. The implementation of this to metrology was first proposed by Caves (1981) [2] in which he employed the use of the squeezed vacuum state $|z;0\rangle \equiv \hat{S}(z)|0\rangle$, this technique has been subsequently optimized [3] to give the precision scaling of the Heisenberg limit.

As squeezing and entanglement both have useful attributes to quantum metrology, this motivates the present work [4] in which we combine the effects and inspect the use within metrology of the "squeezed-entangled state" (SES)

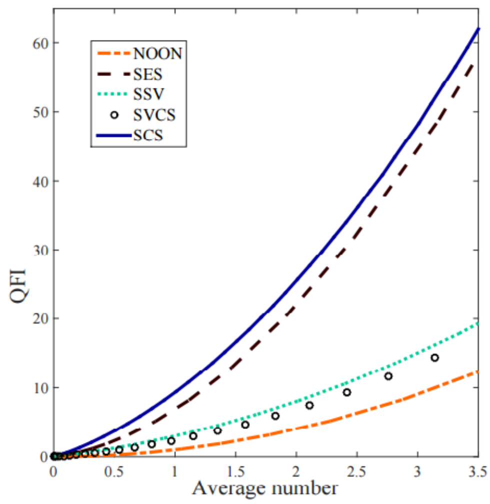
$$|\psi_{\text{SES}}\rangle = \mathcal{N}_{\text{SES}}(|z;0\rangle_a|0\rangle_b + |0\rangle_a|0;z\rangle_b) \quad (1)$$

where, $\mathcal{N}_{\text{SES}} = [2(1+\text{sech}(r))]^{-\frac{1}{2}}$. To determine the potential precision capabilities of a state we use the quantum Fisher information (QFI). For a pure state this is given by $F_Q = 4\left[\langle\psi'(\phi)|\psi'(\phi)\rangle - |\langle\psi'(\phi)|\psi(\phi)\rangle|^2\right]$. This relates to the precision (phase uncertainty) by the Cramèr-Rao bound $\Delta\phi \geq 1/\sqrt{mF_Q}$, where m is the number of experimental repeats. Hence, a large QFI demonstrates high precision capabilities of a state. As shown in the figure to the right, the SES displays a huge improvement over the NOON state with regards to potential precision capabilities.

The main drawback of the SES is that it's difficult to prepare. In order to find a practically viable state which exhibits a phase-estimation potential similar to the SES a particularly promising avenue of investigation is the squeezing of a non-Gaussian state such as superpositions of coherent states (cat states). This motivates the introduction of the squeezed cat state (SCS)

$$|\psi_{\text{SCS}}\rangle = \mathcal{N}_{\text{SCS}} S(z)(|-\alpha\rangle + |\alpha\rangle) \quad (2)$$

where $\mathcal{N}_{SCS} = (2 + 2e^{-2\alpha^2})^{-1/2}$. These states have been generated by Huang *et. al.* [5] with fidelity 67% and size $\alpha = \sqrt{3}$. SCSs may then be used for phase estimation by considering the two-mode state $|\Psi_{\text{SCS}}\rangle = |\psi_{\text{SCS}}\rangle_a \otimes |\psi_{\text{SCS}}\rangle_b$. From the figure below it is evident that the SCS substantially improves over the NOON state and optimal Gaussian state (SSV) [6] for a given average photon number.



Furthermore, we show the SCS is robust enough to maintain a precision advantage over the NOON and SSV states up to 27% loss, thus making it a superior and practical state.

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