

Optical amplifier with coherent photon operation

Ho-Joon Kim,^{1,*} Su-Yong Lee,² Se-Wan Ji,³ and Hyunchul Nha³

¹*School of Computational Sciences, Korea Institute for Advanced Study, Hoegiro 87, Dongdaemun, Seoul 130-722, Korea*

²*Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543 Singapore, Singapore*

³*Science Department, Texas A & M University at Qatar, Education City, P. O. Box 23874, Doha, Qatar*

A deterministic quantum amplifier inevitably adds noise to an amplified signal due to the uncertainty principle in quantum physics. We here investigate how a quantum-noise-limited amplifier can be improved by additionally employing the photon subtraction, the photon addition, and a coherent superposition of the two, thereby making a probabilistic, heralded, quantum amplifier. We show that these operations can enhance the performance in amplifying a coherent state in terms of effective intensity gain, input-output fidelity, and phase uncertainty. In particular, the photon subtraction turns out to be optimal for the fidelity and the phase concentration among these elementary operations, while the photon addition also provides a significant reduction in the phase uncertainty with the largest gain effect.

A deterministic amplification of an unknown optical field has an intrinsic limitation given by the uncertainty principle of quantum mechanics, that says such amplification inevitably accompany with noises added. There have been recent proposals to overcome such quantum limitations in amplification by utilizing probabilistic procedures, and such noiseless amplifiers were proposed to be used in continuous variable quantum cryptography and quantum error correction. We add a realistic way of surpassing the quantum limitations by adding realizable techniques of photon operations to the deterministic linear quantum amplifiers. Photon operations such as photon subtraction and photon addition also have shown various utilities such as entanglement concentration, quantum nonlocality enhancement.

The interaction Hamiltonian of a nondegenerate parametric amplifier used in the deterministic quantum amplifier is given by

$$H_{int} = i\hbar\kappa(a^\dagger b^\dagger - ab), \quad (1)$$

where a and b is the annihilation operator for the signal and the idler mode, respectively. When an input state passes through a nondegenerate parametric amplifier (with vacuum idler mode) and subsequently a photon operation O such as photon subtraction a^m , photon addition $a^{\dagger m}$ or coherent superposition of subtraction and addition $ta + ra^\dagger$, the output state becomes

$$\rho = \frac{1}{N} \int d^2\gamma \frac{1}{\pi(G-1)} \exp\left(-\frac{|\gamma - \sqrt{G}\alpha|}{G-1}\right) O|\gamma\rangle \langle\gamma| O^\dagger, \quad (2)$$

where N is the success probability of the photon operation and G is the gain of the nondegenerate parametric amplifier. The coefficients t and r ($|t|^2 + |r|^2 = 1$) of a coherent photon operation are determined by the transmittance of a beamsplitter and the squeezing parameter both being used to implement the coherent superposition of subtraction and addition.

As a characteristic of an amplifier, we define effective gain as

$$\sqrt{G_e} \equiv \left| \frac{\text{tr}(a\rho_{out})}{\text{tr}(a\rho_{in})} \right|, \quad (3)$$

which quantifies the degree of amplification of the amplitude. Assuming input coherent states having a constant-magnitude amplitude with random uniform phase distribution, for single-photon operations regime, we find that the photon addition gives the largest average gain, while the coherent superposition of subtraction and addition might give the lowest amplification depending on the coefficients t and r .

Next, we examine another characteristic of an amplifier by the fidelity of the output state to an input coherent state, which is defined as

$$F \equiv \langle \sqrt{G}\alpha | \rho_{out} | \sqrt{G}\alpha \rangle. \quad (4)$$

To achieve the same effective gain, photon subtraction turns out to give the highest fidelity while photon addition deteriorates the input coherent state significantly because photon addition always maps classical states into nonclassical ones.

Lastly, we look at the variance of optical phase by the Holevo variance which is used to quantify variance of periodic variables. The Holevo variance can be calculated from the probability distribution function $P(\theta)$ of the experimentally measured values of the optical phase via

$$V = \frac{1}{|\mu|} - 1, \quad (5)$$

where $\mu = \int_0^{2\pi} d\theta P(\theta) e^{i\theta}$ is called the sharpness. The Holevo variance for the ‘canonical measurement’ could be calculated using $\mu = \sum_{n=0}^{\infty} \langle n | \rho | n+1 \rangle$. We find that photon subtraction shows the lowest average Holevo variance for $|\alpha = 0.2\rangle$ while photon addition also achieves relatively lower average variance compared to the deterministic amplifier.

* hojoon.kim@kias.re.kr

[1] Ho-Joon Kim, Su-Yong Lee, Se-Wan Ji, and Hyunchul Nha Physical Review A **85**, 013839 (2012).