Recurrence plots and entropy for the Wigner-function nonclassicality evolution as witnesses of quantum chaotic dynamics of nonlinear Kerr-like oscillator model

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We consider a system of quantum nonlinear oscillator excited by a series of ultra-short coherent pulses [1]. This system is described by the following Hamiltonian

$$\hat{H} = \frac{\chi}{2}\left(\hat{a}^{\dagger}\right)^{2}\hat{a}^{2} + \epsilon\left(\hat{a}^{\dagger} + \hat{a}\right)\sum_{k=1}^{\infty}\delta(t - kT) \qquad (1)$$

where $\hat{a}$ and $\hat{a}^{\dagger}$ are boson annihilation and creation operators, respectively. The parameter χ is the nonlinearity constant, whereas ϵ describes the strength of the interaction between the oscillator and external field. The ultra-short pulses are modeled by a sum of Dirac-delta functions $\delta(t - kT)$ where T denotes the time-interval between two subsequent pulses labeled by k.

We analyze time evolution of non-classicality parameter $\delta(\rho)$ introduced by Kenfack and Życzkowski [3] which depends on the volume of the negative part of the Wigner function $W(\alpha)$. This parameter is defined as:

$$\delta(\rho) = \int \left[|W(\alpha)| - W(\alpha)\right] d\alpha. \qquad (2)$$

$W(\alpha)$ is a special case of the standard Cahil-Glauber s-parametrized quasiprobability distribution (QPD) function $\mathcal{W}^{(s)}(\alpha)$ for $s = 0$ [2], and we apply the later to calculate Wigner function with use of associate Laguerre polynomials. As it was recently shown, such QPD function can be also applied as indicator of nonclassicality of quantum states [4].

We calculate parameter $\delta(\rho)$ for the moments of time just after each external pulse, and reconstruct n-dimensional trajectory describing its evolution. Then, we apply *recurrence plots* analysis [5] to determine whether $\delta(\rho)$ changes its value regularly or in a chaotic way. Moreover, we define entropy-like parameter which helps to evaluate the character of the evolution of $\delta(\rho)$. We examine the situations for various values of ε corresponding to the regular and chaotic evolution of the classical counterpart of our quantum model. We determine the character of changes of our entropic parameter and obtained recurrence plots, when the excitation strength ε changes its value. In consequence, we are able to determine whether quantum chaotic evolution appears in our model or not.

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