

Solving Pauli problem for a photon spatial wave function

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In 1933 Pauli was pondering if a wave function of a particle was uniquely determined by probability distributions of its position and momentum [1]. The problem of reconstruction the complex wave from measurable intensity distributions appears in many branches of physics, e.g. coherence theory, electron microscopy and so forth, so there have been many approaches to solve it. We use Pauli's ideas in quantum optics in conjunction with the Gerchberg-Saxton (GS) phase retrieval algorithm [2–4], which is a tool to compute a wave function from the two intensity measurements.

The techniques are applied to a spatial wave function of a single photons. It is done by performing intensity measurements at two different planes connected by Fourier transform. In our experimental scenario those are image and focal planes of a lens. The algorithm consists of two basic steps. In the first a Fourier transform of a square root of the intensity in the image plane multiplied by a phase from the previous step is computed. In the next step an inverse Fourier transform is evaluated on the square root of the intensity in focal plane multiplied the resulting phase from the previous step. The two steps are repeated iteratively. Couple hundreds of iterations is sufficient to obtain the approximation of the phase.

The experimental setup used for the measurements is presented in Fig. 1(a). An attenuated laser is directed to a system consisting of a polarizer (P), a quarter-wave plate (QWP) and half-wave plate (HWP). Next a photon propagates through birefringent calcite crystal (Altechna,2-BD-CALC-55-1) and is imaged by a system of two lenses L1 and L2. The polarizer P erases distinguishing polarization information of the two possible photon's paths. Measurements in image and focal planes are done using single photon detector (MPD) placed on custom-made motorized XYZ stage. The extracted phase as a result of GS algorithm on the data presented in Fig. 1(b) is depicted in Fig. 1(c).

The method works for certain class of wave functions as the Pauli set composed of the intensity in the image and focal plane is not tomographically complete set [3, 5].

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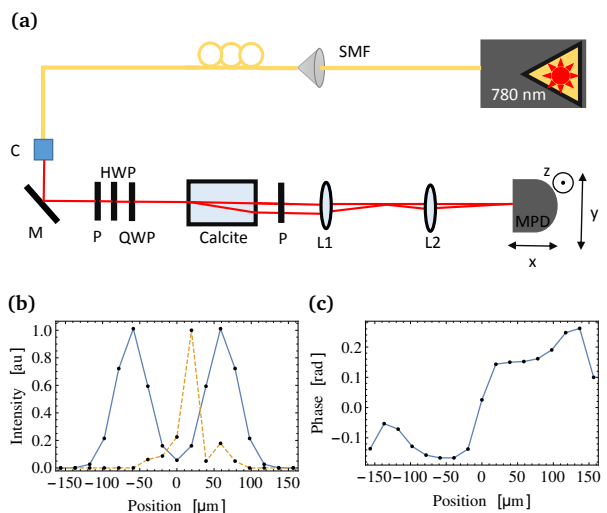


FIG. 1. (a) Experimental setup (SMF - single-mode fiber, C - collimator, M - mirror, P - polarizer, HWP - half-wave plate, QWP - quarter-wave plate, Calcite - calcite crystal, P - polarizer, L1 - lens $f=20\text{cm}$, L2 - lens $f=6\text{cm}$, MPD detector on motorized XYZ stage) (b) Intensity in image (solid line) and focal (dotted line) planes. (c) Recovered phase of spatial wave function.

- [3] J. Corbett, Rep. Math. Phys. **57**, 53 (2006).
 [4] P. Jaming, Appl. Comput. Harmon. Anal. **37**, 413 (2014).
 [5] M. G. Raymer, Contemp. Phys. **38**, 343 (1997).

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- [1] J. R. Fienup, Appl. Opt. **21**, 2758 (1982).
 [2] R. W. Gerchberg and W. O. Saxton, Optik **35**, 237 (1972).