Thermalization in many-particle quantum walks

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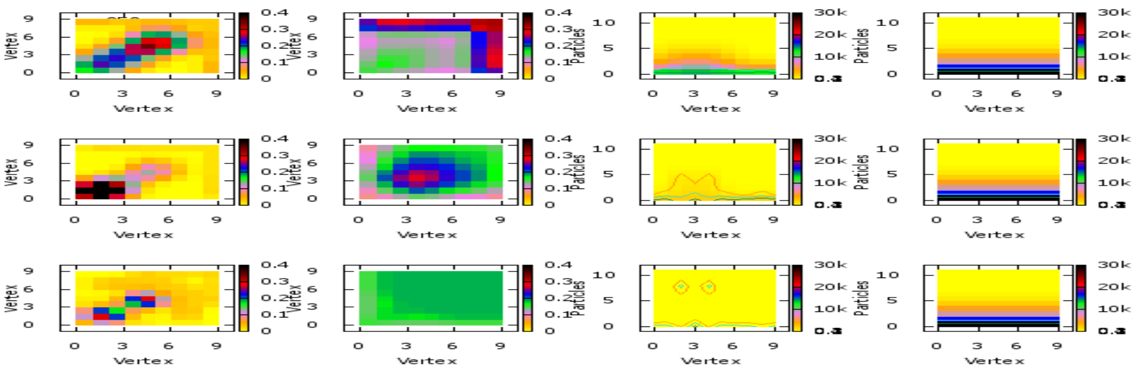
We present many-particle quantum walks on graphs. This study brings together the concepts of many-particles, multimode and indistinguishability to discrete quantum walks. Our scheme is independent of the topology of the graphs: linear, planar or spatial. We use the concepts of position field operators, adjacency matrix and the results of group theory to construct many particle systems on graphs as well as condition shift operators in line with the topology of graphs. The design of our quantum walks associates the coin tossing operation to many quantum walkers at once this is why we also refer to it as shared coin's many-particle quantum walks. Such a dynamics could also be analyzed as quantum walks in the configuration Hilbert space where the word configuration refers to various distributions of quantum walkers on the graph. For a graph of degree d, we have the basis $\{|v_1\rangle, |v_2\rangle, ..., |v_d\rangle\}$ spanning the coin's Hilbert space. These d vectors are also known as chiralities [1] of the coin. The state $|\Psi_r\rangle$

$$|\Psi_r\rangle = \sum_{\ell}\sum_{i} \frac{C^r_{i\vec{\mathbf{n}}_\ell}}{\mathcal{K}^r} |v_i, \vec{\mathbf{n}}_\ell\rangle \qquad (1)$$

is the product of the coin's degrees $|v_i\rangle$ and the M-tuple vectors $\vec{\mathbf{n}}_\ell = (\cdots, \mathbf{n}_\alpha, \cdots)$ which are configurations, where M is the number of vertices on the graph. The $\mathbf{n}_\alpha$'s are position occupation numbers, α is the vertex label. The sum of the $\mathbf{n}_\alpha$'s is constant. The index r indicate the time step. The $C^r_{i\vec{\mathbf{n}}_\ell}$ are amplitudes of configurations and $\mathcal{K}^r$ is the normalization constant. The action of the quantum walks operator contains two stages: coin's tossing performed by means of operator $\mathbf{H}_d = (h_{jk})$ that is a $d \times d$ Hadamard matrix followed by the particle shifting using a field annihilation operator $\widehat{\Psi}(\mathbf{x}_\mu)$ on the vertex μ of origin of the walker and a field creation operator $\widehat{\Psi}^\dagger(\mathbf{x}_\nu)$ on the vertex ν of destination. Since shift takes place on a graph's edge thus is described by the graph adjacency matrix $\mathbf{A} = (A_{\nu\mu})$. Superscript $T(k)$ indicates the direction of the edge. As result operator $\widehat{\mathcal{S}}$ performing the walks takes the form

$$\widehat{\mathcal{S}} = \sum_{\mu,\nu}\sum_{k} \widehat{\Psi}^\dagger(\mathbf{x}_\nu) A^{T(k)}_{\nu\mu} \widehat{\Psi}(\mathbf{x}_\mu) \otimes \sum_{j} h_{jk}|v_j\rangle\langle v_k|. \qquad (2)$$

It is our proposition of the generalization of the shift operator for one-particle quantum walks to the case of many-particle walks on graphs with various topologies. We have $|\Psi_{r+1}\rangle = \widehat{\mathcal{S}}|\Psi_r\rangle$ step implementation. The figures below present the results obtained for the vertex-vertex second order correlations and the vertices counting statistics after 400 steps of simulations of 12 quantum walkers on 10 vertices graphs: the cyclic graph (first row), the torus graph (second row) and the Petersen graph (third row). The first column shows correlations before and the second column after phase transition. The third column contains plots of the particle counting statistics before and the last column after phase transition, respectively.



For all the systems under consideration and for various initial conditions, we observe that when the dimension of the effective Hilbert space reaches value 146860 (or 147070), then it starts to oscillate between these two values during successive time steps. Such behavior is universal and it appears in the counting statistics. Even though the thermalization, the ergodicity and the integrability can explain such a dynamics. We conjecture that the systems universal behavior is due to a transition to integrability of these quantum systems [2]. Research was supported by grant No. DEC-2011/02/A/ST1 /00208 of NSC of Poland.

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[1] A. Ambainis, et al. pages 37–49, 2001.
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