

The theory of quantum filtering was developed by V. P. Belavkin in the 1980's [1, 2] as the extension of classical filtering theory. It has been subsequently developed as a technique for quantum measurement and control [3-5].

We applied the theory of measurement continuous in time based on quantum stochastic calculus of Ito type [6] to derived the filtering equation for a system whose output is either squeezed light or it is mixed with a squeezed light. We considered the situation when the system is driven by a coherent signal and the output field is mixed with a squeezed signal and the case when the input field is taken in a squeezed state. For these two cases the input processes are described by quantum Wiener annihilation processes  $A(\cdot)$  and  $B(\cdot)$  and we have

$$[A(t), A^*(s)] = [B(t), B^*(s)] = t \wedge s,$$

where  $t \wedge s = \min(t, s)$ . The quantum Itô table will then have the form

$$(1) \quad \begin{array}{c|cc} \times & dB & dB^* \\ \hline dB & 0 & dt \\ dB^* & 0 & 0 \end{array}, \quad \begin{array}{c|cc} \times & dA & dA^* \\ \hline dA & mdt & (n+1)dt \\ dA^* & ndt & m^*dt \end{array},$$

where  $n > 1$  and  $|m|^2 \leq n(n+1)$ . Let us note that the squeezed terms rely on the non-Fock (i.e., Araki-Woods) representation and the scattering process is not well-defined in this case.

When the system is driven by a coherent input the unitary evolution operator  $V(t)$  of the compound system (the system and the external field) satisfies the quantum stochastic differential equation of the form

$$(2) \quad dV(t) = \left\{ L \otimes dB^*(t) - L^* \otimes dB(t) - \left( \frac{1}{2} L^* L + iH \right) \otimes dt \right\} V(t)$$

with  $V(0) = 1$ . Here  $L$  is bounded operators and  $H$  self-adjoint. We derived the posterior evolution conditioned by results of simultaneous measurement of quadratures

$$Y_1(t) = \frac{1}{\sqrt{2}} (B^{\text{out}}(t) + B^{\text{out}*}(t) + A(t) + A^*(t)),$$

$$Y_2(t) = \frac{1}{\sqrt{2}i} (B^{\text{out}}(t) - B^{\text{out}*}(t) - (A(t) - A^*(t))).$$

Here  $B^{\text{out}}(t) = V^*(t)B(t)V(t)$  and it is called the output process.

As an example we took the system to be a single cavity mode in a Gaussian state. We proved that when the system is initially in a Gaussian state it remains in a Gaussian state. We found the expressions for the conditional mean values of the observables of the system and we encountered the conditional dispersions of optical quadrature of the system. Moreover, we gave the answer to the question when the filtering equation transforms pure state into pure ones. In general, when the measurement is imperfect pure states are not preserved.

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