New Developments in the Stroboscopic Tomography

Artur Czerwiński[1,*]

[1] *Institute of Physics, Nicolaus Copernicus University in Toruń, Poland*

According to one of the most fundamental assumptions of quantum theory, the density matrix carries the achievable information about the quantum state of a physical system. In recent years the determination of the trajectory of the state based on the results of measurements has gained new relevance because the ability to create, control and manipulate quantum states has found applications in other areas of science, such as: quantum information theory, quantum communication and computing.

In this work we follow the stroboscopic approach to quantum tomography which was proposed in [1] and then developed in [2, 3]. In the stroboscopic approach we consider a set of observables $\{Q_i\}_{i=1}^r$ with $r < N^2 - 1$ (where $N = dim\mathcal{H}$) and each of them can be measured at time instants $\{t_j\}_{j=1}^p$. Every measurement provides a result that shall be denoted by $m_i(t_j)$ and can be represented as $m_i(t_j) = Tr(Q_i\rho(t_j))$. Because in this approach the measurements are performed at different time instants, it is necessary to assume that the knowledge about the character of evolution is available, e.g. the Kossakowski-Lindblad master equation [4] is known or, equivalently, the collection of Kraus operators. Knowledge about the evolution makes it possible to determine not only the initial density matrix but also the complete trajectory of the state. To make this issue clearer from now on we assume the following definition [3].

Definition 1.
An N-level open quantum system is said to be $(Q_1, ..., Q_r)$-reconstructible on an interval $[0, T]$ if there exists at least one set of time instants $\{t_j\}_{j=1}^p$ ordered as $0 \leq t_1 < ... < t_p \leq T$ such that the trajectory of the state can be uniquely determined by the correspondence

$$[0, T] \ni t_j \rightarrow m_i(t_j) = Tr(Q_i\rho(t_j)) \tag{1}$$

for $i = 1, ..., r$ and $j = 1, ..., p$.

The outcomes that we obtain from the measurements can be presented in a matrix form as

$$\begin{bmatrix} m_1(t_1) & m_1(t_2) & \cdots & m_1(t_p) \\ m_2(t_1) & m_2(t_2) & ... & m_2(t_p) \\ \vdots & \vdots & \ddots & \vdots \\ m_r(t_1) & m_r(t_2) & \cdots & m_r(t_p) \end{bmatrix}. \tag{2}$$

The fundamental question that we formulate is: *Can we reconstruct the initial density matrix $\rho(0)$ for a given master equation from the set of measurement results presented in* (2)*?*

Other questions that arise in this approach concern: the minimal number of observables for a given master equation and their properties as well as the minimal number of time instants and their choice. The general conditions for observability have been determined and the proofs can be found in papers [1–3].

In the this work [5] there are three different decoherence models of 2-level quantum systems, to which the stroboscopic approach has been applied. In the static approach to quantum tomoraphy of 2-level systems one needs to measure three different observables to be able to reconstruct the initial state. In this work we discuss effectiveness of the stroboscopic approach in comparison with the static model.

First, we address the problem of dephasing, in which the stroboscopic approach allows us to give the concrete formula for the initial density matrix. The required number of different observables is equal 2, which gives the stroboscopic approach an advantage over the static model.

Next we discuss the usefulness of the stroboscopic approach in case of depolarization, which is another model of decoherence. The stroboscopic tomography does not allow us to decrease the number of observables in this case. Therefore, this approach seems as effective as the static one.

Finally, we tackle a more general problem, where the stroboscopic approach seems to have the greatest advanatage. As the most promising result we introduce a parametric-dependent family of Kraus operators for which the generator of evolution has no degenerate eigenvalues, i.e. in that case there exists one observable the measurement of which performed at three different instants is sufficient to reconstruct the initial density matrix.

ACKNOWLEDGEMENT

This research has been supported by grant No. DEC-2011/02/A/ST1/00208 of National Science Center of Poland.

* aczerwin@fizyka.umk.pl

[1] A. Jamiołkowski, Int. J. Theor. Phys. **22**, 369 (1983).
[2] A. Jamiołkowski, Rep. Math. Phys. **46**, 469 (2000).
[3] A. Jamiołkowski, OSID **11**, 63 (2004).
[4] V. Gorini, A. Kossakowski,E. C. G. Sudarshan, J. Math. Phys. **17**, 821 (1976).
[5] This work is based on the article that has been submitted for publishing in IJTP and is currently undergoing the review process.