Study of unitary transform for subadditivity condition

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The quantum correlations in bipartite systems, e. g. in two-qubit systems are responsible for such [1, 2] phenomenon as entanglement. The quantum correlations can be also characterized by different entropic and information inequations. For example the subadditivity condition relate to von Neumann entropy of the composite system containing two subsystems with the entropies of the subsystem states. The phenomenon of entanglement (or separability) is described by the properties of composite system density matrix.

These properties are related with possibility to present the density matrix in the form of convex sum of the tensor products of the density matrix of the subsystem states. The separable states have the density matrix which can be presented in such a form. The entangled states correspond to density matrices of composite system which can not be presented in such form. On the other hand, this property depends on the reference frame. If new reference frame in the Hilbert space of states is changed by means of local unitary transform the separability property of density matrix of composite system is reserved. The global unitary transform of the density matrix violates the separability property. In the present work we study the quantum correlation in the noncomposite system investigating its density matrix. We focus on different kinds of entropic inequations introducing two artificial subsystem density matrices.

The aim of the work is to obtain the subadditivity condition which is entropic-information inequality for single qudit state and study dependence of the quantum information in the form of unitary transform of the density matrix of the qudit state. We follow in our study the approach used in [3, 4].

We consider on example of qudit state with $j = 3/2$ how the entropic inequality which is subadditivity condition can be written for the density matrix of the qudit state as the function of the matrix elements of the unitary 4x4 matrix. The "local" and "global" unitary transforms provide different behavior of the introduced quantum information associated with the single qudit state. The physical meaning of the obtained result needs extra clarification and will be discussed in future publications.

For example, we consider the unitary transform with corresponding matrix in the form

$$U = \exp\left(ih\sum_{i=1}^{15} n_i \tau_i\right)$$

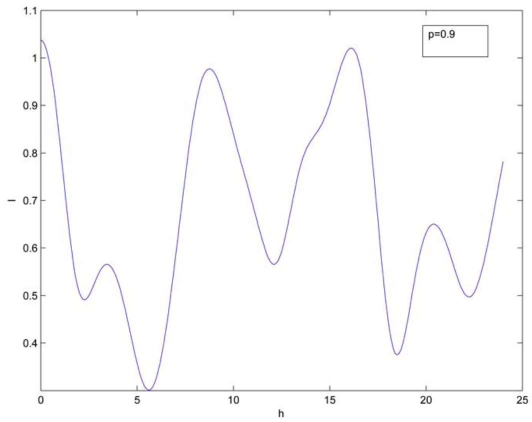

FIG. 1. The dependence of quantum mutual information I on parameter of unitary transform h. In initial moment system was in the Werner state with $p = 0.9$.

where τ_i is an element of su(4) algebra and $\sum_{i=1}^{15} n_i^2 = 1$. So, varying parameter h we can get different unitary matrices and for each of them we calculate density matrix of transformed state and quantum information

$$I = Tr\rho\ln\rho - TrR_1\ln R_1 - TrR_2\ln R_2,$$

here ρ is transformed density matrix and R_1 and R_2 are density matrices which can be obtained via operation of partial trace.

One of our results is shown on Fig. 1.

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[1] E. Schrödinger, *Naturwissenschaften*, **23**, 807–812; 823–828; 844–849 (1935).
[2] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **44**, 777 (1935).
[3] V.I. Man'ko, O.V. Man'ko, JETP, **85 (3)**, 430 (1997).
[4] V.I. Man'ko and L.A. Markovich, J. Russ. Laser Res., **35 (2)**, 200 (2014)