What is Quantum non-Markovianity?

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The general dynamics for a closed quantum system is unitary, completely determined by the Shrödinger equation. The quantum systems we are interested in are open, i.e., coupled to an environment. By tracing over the environment, one can describe the system evolution by a superoperator, the dynamical map, which maps the intial quantum state of the system to the state at some later time.

One fundamental problem in this field is distinguishing two classes of evolution of open systems: Markovian and non-Markovian. This distinction is well understood in the classical case: a stochastic process is Markovian if the future statistics only depend on the present state, independently of its history. In particular, if the classical system state takes the values $x_0, x_1, \ldots, x_n$ at successive times $t_0, t_1, \ldots, t_n$, then the stochastic process is Markovian if and only if the probability of the state being x at any later time t satisfies $P(x,t|x_n,t_n;...;x_0,t_0) = P(x,t|x_n,t_n)$.

Clearly, since quantum states are described by density operators rather than classical random variables, one must take another approach to define Markovianity for quantum evolution processes. However, there is no general consensus as to the 'correct' approach: different people use the term "quantum non-Markovianity" to mean very different things! Hence, to avoid confusion, we will avoid attributing any definite meaning to this term. Instead, we aim to significantly clarify the various issues, by discussing a large number of concepts that have been, or could reasonably be, used to define quantum Markovianity, and proving a number of hierarchical relations between them (Fig. 1).

For example, one approach to quantum Markovianity is based on the idea that the system-environment interaction has almost no effect on the state of the (typically) much larger environment. This leads naturally to the Born approximation (or weak coupling limit), in which the combined state is approximated by a factorisable one, and is well satisfied in many cases in quantum optics [1]. An alternative approach is based on "quantum white noise" in which the environment can always be divided in to the past (interacted) and the future (yet-to-interact) components. Both approaches are stronger than requiring the quantum regression formula (QRF) for multitime system correlations to be valid [3], which is in turn stronger than the requirement of "composability" of the dynamical map. This last concept corresponds to being able to represent the system evolution as two successive evolutions without resetting the system-environment interaction.

The dotted border in Fig. 1 demarcates the above approaches, based on properties of the system-environment interaction, from approaches to quantum Markovianity

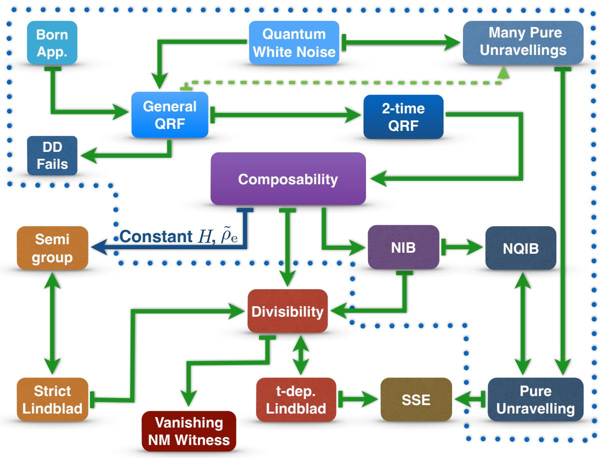

FIG. 1. Hierarchy of open quantum system dynamics

based solely of properties of the dynamical map. For example, "divisibility", motivated by the classical Chapman-Kolmogorov equation [1], assumes that the dynamical map for a given evolution time can be obtained by application of a completely positive map to the dynamical map at any earlier time [2], and is equivalent to the existence of a (time-dependent) Lindblad-type master equation. Other approaches of this type include requiring the dynamical map at different times to form a semigroup.

We also prove relations between these and other properties of interest for open quantum systems, such as the applicability of dynamical decoupling to preserve quantum information, the existence of (quantum) information backflow from the environment, and the physical reality of stochastic pure-state trajectories (Fig. 1). We argue that all these concepts are related to or reflect the notion of quantum Markovianity. Finally, we indicate the complexity of this notion by considering analogous approaches for the classical case – for which many of the approaches in Fig. 1 either coalesce or become inapplicable.

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[1] C. Gardiner and P. Zoller, Quantum Noise (Springer, Berlin, 2004).

[2] A. Rivas, S. F. Huelga and M. B. Plenio, Phys. Rev. Lett. 105, 050403 (2010).

[3] N. L. Gullo, I. Sinayskiy, T. Busch, and F. Petruccione, arXiv preprint arXiv: 1401.1126 (2014).