Three-mode nonlinear state truncation – squeezing, quantum correlations and entanglement

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We consider a system of three Kerr-like quantum oscillators forming a sequence of subsystems and labeled by 1, 2 and 3. The oscillators are coupled each other and additionally, are excited by two modes of external coherent fields. The Hamiltonian describing our model can be written in the following form:

$$\hat{H} = \hat{H}_{nl} + \hat{H}_i + \hat{H}_e, \tag{1}$$

where

$$\hat{H}_{nl} = \frac{\chi}{2}\left(\hat{a}_1^\dagger\right)^2 \hat{a}_1^2 + \frac{\chi}{2}\left(\hat{a}_2^\dagger\right)^2 \hat{a}_2^2 + \frac{\chi}{2}\left(\hat{a}_3^\dagger\right)^2 \hat{a}_3^2 \tag{2}$$

describes "free" evolution of the oscillators,

$$\hat{H}_i = \left(\varepsilon \hat{a}_1^\dagger \hat{a}_2 + \varepsilon^* \hat{a}_2^\dagger \hat{a}_1\right) + \left(\varepsilon \hat{a}_2^\dagger \hat{a}_3 + \varepsilon^* \hat{a}_3^\dagger \hat{a}_2\right) \tag{3}$$

represents the interaction between two neighboring oscillators, whereas

$$\hat{H}_e = \alpha \hat{a}_1^\dagger + \alpha^* \hat{a}_1 + \alpha \hat{a}_3^\dagger + \alpha^* \hat{a}_3 \tag{4}$$

corresponds to interaction with external coherent field. We assume here that the nonlinearity constants χ, characterizing three oscillators, are identical. Moreover, interaction strengths between modes $1-2$ and $2-3$ are described by the same parameter ε, whereas external excitations in the modes 1 and 3 by α.

We show that for weak excitation regime, our model behaves as three–mode *nonlinear quantum scissors* [1] exhibiting *photon blockade* [2] effect. In consequence, its time-evolution remains closed within a set of 8 states $|i\rangle \otimes |j\rangle \otimes |k\rangle = |i\rangle|j\rangle|k\rangle$ where $\{i, j, k\} = \{0, 1\}$, and our system can be treated as *three-qubit* model.

For such situation we derived analytical formulas for the probability amplitudes corresponding to these states, allowing us to calculate various parameters describing quantumness and quantum correlations present in our model. In particular, there are $g^{(1)}(t)$ and $g^{(2)}(t)$ functions corresponding to the correlations not only in each of three modes of the field but also to the intermode correlations. We also find and discuss parameters describing squeezing effects and finally, two- and three-partite entanglement. For instance, we show that not only Bell-like states can be generated but also various classes of three-qubit entangled states can appear in our system.

Moreover, we discuss the dynamics of our system influenced by external bath and consider amplitude and phase damping processes. We show that under some conditions sudden entanglement death [3–5] and its rebirth [6–8] can appear in the system, and compare the differences between the behaviors of bi- and three-partite entanglement.

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