Entangled coherent states for the description of the interaction of matter and radiation

R. López-Peña,[1] S. Cordero,[1] E. Nahmad-Achar,[1] and O. Castaños[1]

[1]*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México*
Apartado Postal 70-543, 04510 México DF, México

We will use superposition of coherent states for the description of the ground and first excited states of the Dicke model and its generalisation to three levels.

First we consider the Dicke Hamiltonian for N_A two-level atoms [1]

$$\hat{H}_2 = \Omega\,\hat{a}^\dagger\hat{a} + \omega_A\,\hat{J}_z - \frac{\mu}{\sqrt{N_A}}\left(\hat{a}^\dagger + \hat{a}\right)\left(\hat{J}_+ + \hat{J}_-\right) .$$

In this expression $\hat{a}^\dagger$ and $\hat{a}$ denote the creation and annihilation operators of the one-mode radiation field of frequency Ω, ω_A the separation between the atomic levels, $\hat{J}_z$ denotes a collective atomic operator which counts the difference of population between the two atomic levels, $\hat{J}_+$ a collective atomic operator which promotes atoms from the lower level to the upper level, $\hat{J}_- = \hat{J}_+^\dagger$, and μ stands for the dipolar intensity in the interaction between matter and radiation.

A variational description of the ground state of $\hat{H}_2$ can be achieved using as a trial state the tensorial product of $HW(1)$ coherent states for radiation field and $SU(2)$ coherent states for the atoms. Minimizing with respect to the parameters of the coherent states we obtain a good approximation of the ground state of the system. [2]

However the Dicke Hamiltonian is invariant under the transformation by the unitary operator $\hat{U} = \exp\left(i\,\pi\,\hat{M}\right)$, where we have introduced the excitation number operator

$$\hat{M} = \hat{a}^\dagger\hat{a} + \hat{J}_z + \frac{N_A}{2} .$$

This symmetry is not present in the variational approximation of the ground state by $HW(1)\otimes SU(2)$ coherent states. A better description is obtained using the projected states obtained when we consider states with even or odd components of $\hat{M}$. In this manner we are able to obtain an approximation not only for the ground state but also for the first-excited state. Because the expectation value of the Hamiltonian in these states is more complicated to minimize, we use the minima obtained for the $HW(1)\otimes SU(2)$ coherent states. Our *even and odd states* have analytical forms in terms of the model parameters and allow us to calculate closed expressions for the expectation values of field and matter observables. [3]

Next we apply this idea to the generalisation of the Dicke model to three-level atoms interacting with one-mode radiation

$$\hat{H}_3 = \Omega\,\hat{a}^\dagger\hat{a} + \sum_{j=1}^{3}\omega_j\,\hat{A}_{jj} - \sum_{1\leq j<k\leq 3}\frac{\mu_{jk}}{\sqrt{N_A}}\left(\hat{a}^\dagger + \hat{a}\right)\left(\hat{A}_{jk} + \hat{A}_{kj}\right) .$$

where $\hat{A}_{ij}$ denotes the collective matter operators which moves atoms from i-th level to j-level if $i \neq j$ and counts the number of atoms in the i-level if $i = j$, the atomic levels are ordered such that $\omega_1 \leq \omega_2 \leq \omega_3$, and μ_{ij} denote the dipolar intensities. A particular atomic configuration is set by properly choosing one dipolar parameter $\mu_{ij} = 0$.

If we suppose a system of identical atoms we can consider for the atomic part the coherent state

$$|N_a;\gamma\rangle = \frac{1}{\sqrt{N_a!}}\left[\hat{\Gamma}^\dagger(\gamma)\right]^{N_a}|0\rangle ,$$

where we defined the operator

$$\hat{\Gamma}^\dagger(\gamma) := \frac{1}{\sqrt{\sum_k|\gamma_k|^2}}\sum_{j=1}^{3}\gamma_j\,\hat{b}_j^\dagger ,$$

and the creation $\hat{b}_i^\dagger$ and annihilation $\hat{b}_j$ operators with i, $j = 1, 2, 3$ satisfy the commutation relations

$$\left[\hat{b}_i,\,\hat{b}_j^\dagger\right] = \delta_{ij}\,\hat{1} ,\ \left[\hat{b}_i,\,\hat{b}_j\right] = 0 = \left[\hat{b}_i^\dagger,\,\hat{b}_j^\dagger\right] .$$

As before we can use as trial states the tensor product of $HW(1)$ and $SU(3)$ coherent states and get a good approximation for the ground state of the considered configuration, but again the state does not preserve the symmetry present in the Hamiltonian. In these case the symmetry operator is again of the form $\hat{U} = \exp(i\,\pi\,\hat{M})$, where $\hat{M}$ is again a excitation number operator and has the form

$$\hat{M} = \hat{a}^\dagger\hat{a} + \lambda_2\hat{A}_{22} + \lambda_3\hat{A}_{33} ,$$

where λ_i, $i = 2, 3$, depends on the configuration of the levels considered. Using the superposition of states adapted to the symmetry, i.e., states with only *even or odd components* of $\hat{M}$, and the solutions obtained for the normal coherent states $HW(1)\otimes SU(3)$, we get analytical expressions for the two states lowest in energy and are able to calculate closed forms for the expectation values of the observables of the system. [4]

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