Experimental Violation of Multisetting Bell Inequalities for Qudits using Energy-Time Entangled Photons

S.Schwarz,[1,*] B.Bessire,[1] Y.-C. Liang,[2] and A. Stefanov[1]

[1]*Institute of Applied Physics, University of Bern, 3012 Bern, Switzerland*
[2]*Department of Physics, National Cheng Kung University, Taiwan*

The non-locality of nature is one of the most profound notions of quantum mechanics (QM) and is closely related to the phenomena of entanglement. The latter occurs when a quantum state of at least two particles cannot be written as independent single particle states. The manifestation of entanglement can be observed by correlations between two systems with no classical analogue. Moreover, these correlations are capable to violate linear constraints on experimental input-output correlations derived by means of a local hidden variable (LHV) model. Such constraints are well-known as Bell inequalities [1]. Experimentally, a large number of such Bell measurements have already been performed, mostly with $d = 2$ dimensional states, i.e. qubits, and dichotomic measurement settings and outcomes [2]. The experimental setup we use is capable of generating and manipulating entangled d-dimensional quantum states, denoted as qudits. This allows to study multisetting Bell inequalities which allow for deeper insights into the nature of non-locality and entanglement than their 2-dimensional predecessors.

A generic multisetting Bell experiment involves two parties Alice (A) and Bob (B) performing independent measurements A_x and B_y with settings $x \in \{0,1,...,s_A\}$ and $y \in \{0,1,...,s_B\}$ on a shared two-qudit state. This yields measurement outcomes $a \in \{0,1,...,o_A\}$ for Alice and $b \in \{0,1,...,o_B\}$ for Bob. In this way, a set of joint-conditional probabilities $P(a,b|x,y)$ determines a Bell parameter $I^{s_A s_B}_{oAoB}$ and the corresponding Bell inequality then reads $I^{s_A s_B}_{oAoB} < B$. The constant B denotes the LHV limit. The simplest Bell inequality for $d = 2$ systems is the Clauser-Horne-Shimony-Holt (CHSH) inequality [3] where $s_A = s_B = 2$ and $o_A = o_B = 2$. A first generalization of the CHSH inequality to d-dimensional states was carried out by Collins-Gisin-Linden-Massar-Popescu (CGLMP) [4] and experimentally implemented for maximally and non-maximally entangled qutrits in [5]. Numerical investigations, however, indicate, that, opposed to CGLMP, there exist multisetting Bell inequalities where a maximal violation is obtained for $o_A = o_B \neq d$ [6]. Here, we numerically and experimentally study the case where $o_A = o_B > d$. In our experiment we discretize the spectrum of energy-time entangled photons into bins with a spatial light modulator (Fig. 1). The latter allows to implement with a high degree of flexibility the large number of projective measurements needed to study the properties of multisetting Bell inequalities. Figure 2 shows I^{33}_{33} for a 2-qubit state

$$|\psi(\gamma)\rangle = \frac{1}{\sqrt{1+\gamma^2}}(|0\rangle_A|0\rangle_B + \gamma|1\rangle_A|1\rangle_B),$$

with a variable degree of entanglement $\gamma \in [0,1]$.

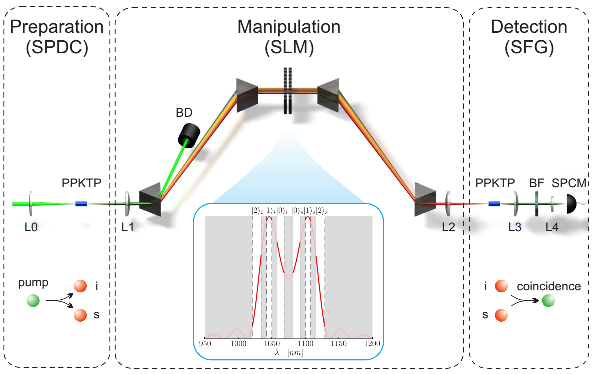

FIG. 1. Schematic view of the experimental setup. *Preparation*: Energy-time entangled two-photon states are prepared via spontaneous parametric down-conversion (SPDC) by using a nonlinear PPKTP crystal. *Manipulation*: Their spectrum is shaped in amplitude and phase by a spatial light modulator (SLM). The inset shows a simulated SPDC spectrum discretized into frequency bins (parts in gray refer to pixels of zero transmission). *Detection*: The coincidence signal is generated via sum-frequency generation (SFG) and measured with a single photon counting module (SPCM).

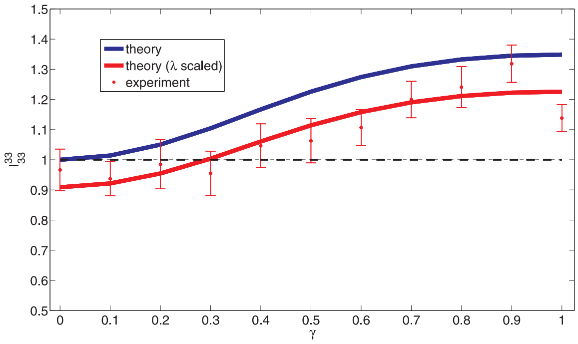

FIG. 2. Bell parameter I^{33}_{33} for two-qubit state with variable degree of entanglement γ. By means of a symmetric noise model the theoretical curve (blue) is scaled onto the data (red curve).

* sacha.schwarz@iap.unibe.ch

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