Properties of 3-Level Systems: the triple phase transition, and critical exponents

Eduardo Nahmad-Achar,[1,*] Sergio Cordero,[1] Octavio Castaños,[1] and Ramón López-Peña[1]

[1]*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México*
Apdo. Postal 70-543, México D.F. 04510

The phase diagram structure is studied for 3-level atomic systems in the presence of a one-mode radiation field. In particular, the energy spectrum and its degeneracies is studied for the Ξ-configuration, as well as the presence of a *triple point*, i.e., a coexistence of systems with 3 different values of the total number of excitations [1]. This triple point is fixed in parameter space, is present for any finite number of atoms N_a, and prevails in the thermodynamic limit. This means that in a vicinity of this point any quantum fluctuation will drastically change the composition of the ground state of the system. The expectation value of the atomic population of each level, that of the number of photons, and the probability distribution of photons at the triple point, as well as the state structure, are all obtained.

The existence of this triple point in phase space, which is independent of the number of atoms, is a characteristic of the Ξ-configuration; it does not appear in the Λ or the V configurations. We show that the fidelity susceptibility is a much more sensitive function than the fidelity itself to find the transitions in parameter space. The density of energy levels and the energy spectra at different values of the total excitation number are also studied. In particular, we show the collapse, in the limit $N_a \to \infty$, of the successive energy levels for each value of the excitation number at the triple point.

The behaviour of the important observables of a system near phase transitions may be described by the system's critical exponents, and these are believed to be universal with respect to physical systems. We therefore derive the critical exponents for the system of 3-level atoms. Since in the collective region we have a decay rate proportional to the number of atoms squared, as opposed to the number of atoms for the normal region, and since important observables of the system change significantly in a very small vicinity of the parameter space, this gives hope for experimental exploitation both at the triple point and elsewhere near the quantum phase transitions.

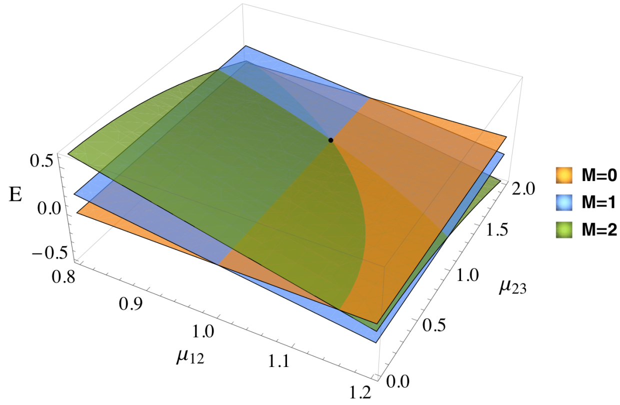

FIG. 1. Energy of the ground state plotted as a function of μ_{12} and μ_{23} for the Ξ-configuration in double resonance. The 3 regions meet at a point, the *triple point*, at coordinates $(1, \sqrt{2}, 0)$ in parameter space (marked in the figure with a black dot). This point is independent of the number of atoms, and subsists in the thermodynamic limit. M is the total number of excitations of the system.

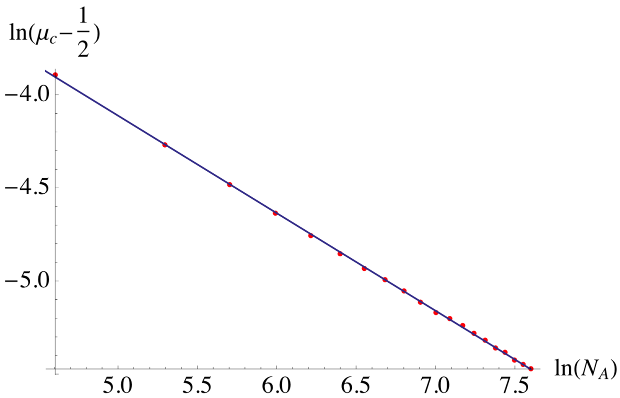

FIG. 2. Logarithmic behaviour of the critical value of the coupling parameter μ_c with the number of atoms N_A, for the symmetry-adapted (SAS) ground state in the V-configuration. The linear fit (continuous straight line) shows a critical exponent of $-11/21$, exactly as that found for the Dicke model using SAS states.

* nahmad@nucleares.unam.mx

[1] E. Nahmad-Achar, S. Cordero, R. López-Peña and O. Castaños, *J. Phys. A: Math. Theor.* **47** 455301 (2014).

[2] E. Nahmad-Achar, S. Cordero, O. Castaños and R. López-Peña, "Phase Diagrams of Systems of 2 and 3 levels in the presence of a Radiation Field" *Phys. Scr.* (2015) (in press).