Non-Markovianity and the flow of information

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To understand the information flow between a system and it environment, we based our studies on the decoherence program [1]where a system $\mathcal{S}$ is coupled to a measurement apparatus $\mathcal{A}$, which in turn interacts with an environment $\mathcal{E}$. In this sense, it introduces the concept of assisted knowledge, where the environment $\mathcal{E}$, acquires information about a system $\mathcal{S}$, by means of its measurement apparatus $\mathcal{A}$.

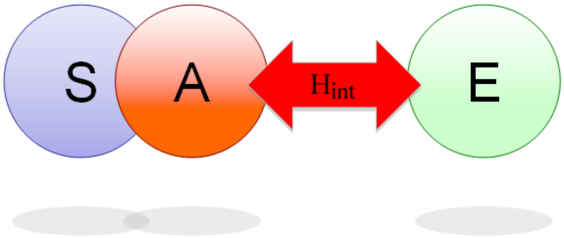

FIG. 1: Schematic representation of the assisted knownledge where the environment $\mathcal{E}$ acquires information about a system $\mathcal{S}$ by means of an interaction with apparatus $\mathcal{A}$.

In quantum information theory there are different ways to measure information and leakage when considering open quantum systems. Here, we focus in *quantum loss* [2] and *classical correlation* [3]. Quantum loss L represents the amount of information that is getting lost in the environment $\mathcal{E}$. If we consider that initially $\mathcal{SAE}$ is given by a pure state the quantum loss is measure by the mutual information between system and environment, i.e. it measures the total correlation that system $\mathcal{S}$ shares with $\mathcal{E}$,

$$L = S(\rho_{\mathcal{S}}) + S(\rho_{\mathcal{E}}) - S(\rho_{\mathcal{SE}}), \tag{1}$$

where $S(\rho) = -\mathrm{Tr}(\rho \log_2 \rho)$ is the von-Neumann entropy.

On the other hand, classical correlation $J^{\leftarrow}_{\mathcal{SE}}$ represents the maximum amount of classical information that can be extracted about the system $\mathcal{S}$ through the observation of the environment $\mathcal{E}$. It is given by

$$J^{\leftarrow}_{\mathcal{SE}} = \max_{\{\Pi_i^{\mathcal{E}}\}} \left[S(\rho_{\mathcal{S}}) - \sum_i p_i S(\rho_{\mathcal{S}|i}) \right], \tag{2}$$

where $\{\Pi_i^{\mathcal{E}}\}$ represents the general quantum measurements (including the non-orthogonal ones) acting on the environment $\mathcal{E}$ [3] and $\rho_{\mathcal{S}|i} = \mathrm{Tr}_{\mathcal{E}}(\Pi_i^{\mathcal{E}} \rho_{\mathcal{SE}} \Pi_i^{\mathcal{E}})/p_i$ denotes the remaining state of the subsystem $\mathcal{S}$ after obtaining the outcome i with probability $p_i = \mathrm{Tr}_{\mathcal{SE}}(\Pi_i^{\mathcal{E}} \rho_{\mathcal{SE}} \Pi_i^{\mathcal{E}})$ in the subsystem $\mathcal{E}$.

In our work, employing the concept of assisted knowledge, and the definition of quantum loss and classical correlation we show how to understand the backflow of information from the environment to the system for two distinct measures of non-Markovianity: entanglement and mutual information-based measures. Entanglement-based measurement was introduced by Rivas, Huelga, and Plenio (RHP) [4] and it is grounded on the fact that the entanglement between two subsystems does not increase by means of local operations. In the same manner, as the entanglement-based measure, the mutual information-based measure [5], presented by Luo, Fuo, and Song (LFS), is based on the fact that mutual information between two subsystems does not increase by means of local operations. Despite similar, these measures are conceptually different and our result present an interpretation for both in terms of flow of information.

The first interpretation, related to the entanglement-based measure of non-Markovianity, is given by a simple relation between the rate of changes of the entanglement of formation and the accessible information (classical correlation) [6],

$$\frac{d}{dt} E_{\mathcal{SA}} = -\frac{d}{dt} J^{\leftarrow}_{\mathcal{SE}}. \tag{3}$$

This relation immediately implies that any temporary decrease in $J^{\leftarrow}_{\mathcal{SE}}$ will be reflected as a temporary increase of $E_{\mathcal{SA}}$. Thus, the non-Markovianity measure based on the rate of change of the accessible information $J^{\leftarrow}_{\mathcal{SE}}$ can also be expressed in terms of the rate of change of the entanglement of formation between the system $\mathcal{S}$ and the apparatus $\mathcal{A}$ [6]. According to the RHP criterion, any temporary revival of entanglement is an indication of the non-Markovian nature of a quantum process since that classical correlation gives an interpretation to the RHP measure in terms of information flow.

The second interpretation, related to the mutual information-based measure of non-Markovianity, is given by the relation [7]

$$\frac{d}{dt} L = -\frac{d}{dt} I_{\mathcal{SA}}. \tag{4}$$

Recalling that the LFS measure [5] of non-Markovianity is based on the rate of change of the quantum mutual information shared by the system $\mathcal{S}$ and the apparatus $\mathcal{A}$, we immediately realize that in fact the quantum loss approach to non-Markovianity is exactly equivalent to the formulation of the LFS measure. Indeed, since the LFS measure captures the non-Markovian behavior through a temporary increase of the mutual information of the bipartite system $\mathcal{SA}$, the quantum loss gives an interpretation to the LFS measure in terms of information exchange between the system $\mathcal{S}$ and the environment $\mathcal{E}$, since any temporary loss of L will be observed as a temporary revival of $I_{\mathcal{SA}}$ by the same amount.

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