

# Creation of superposition of unknown quantum states

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The existence of superpositions of pure quantum states is one of the most intriguing consequences of the postulates of quantum mechanics. Quantum superpositions are responsible for numerous nonclassical phenomena that are considered to be the key features of quantum theory [1]. The prominent examples are: quantum interference [2–4] and quantum entanglement [5]. Superpositions of pure states are not only important from the foundational point of view but also a feature of quantum mechanics that underpins the existence of ultrafast quantum algorithms (such as Shor factoring algorithm [6] or Grover search algorithm [7]), quantum cryptography and efficient quantum metrology [8].

The importance of quantum superpositions provokes questions about the limitations that quantum mechanics itself imposes on the possibility of their generation. In this work we systematically study the problem of creation of superpositions of unknown quantum states. We consider a scenario in which we are given two particles in unknown pure quantum states and our task is to create, using operations allowed by quantum mechanics, superposition of these states with some given complex weights  $\alpha, \beta$  satisfying  $|\alpha|^2 + |\beta|^2 = 1$ . Our work consists of three parts in which we tackle various aspects of this problem.

In the first part, inspired by the known no-go results in quantum mechanics (such as no-cloning [9], no-broadcasting [10] or no-deleting theorem [11]), we formulate and prove a no-go theorem that forbids the existence of general quantum operations producing a superposition of two unknown quantum states. By “general quantum operations” we understand any quantum channel followed by the postselection conditioned on the result of some generalized measurement. In our formulation of the no-go theorem we impose minimal assumptions on the out-

come of the quantum operation. First of all we assume that the superpositions are created only probabilistically and that for certain input states the probability of success can equal zero. Secondly, we assume that for each pair of input states the protocol produces the superpositions of the input states for some choice (in general depending on the input states) of the relative phase between vectors appearing in the superposition.

In the second part we present an explicit protocol that probabilistically generates superpositions of two unknown pure states, each having a fixed overlap with the known referential pure state. Moreover, we show that the protocol described by us is unique and thus also optimal for the considered scenario. The crucial role in our protocol is played by the conditional swap operation and our construction is valid for the arbitrary dimension of the considered Hilbert space. The protocol derived by us can be used to engineer nonclassical multi-mode states of light (out of coherent or Gaussian states having a fixed overlap with the vacuum) or to generate entangled states in the optical implementations of multipartite systems.

In the last part we study the optimal approximate generation of quantum superpositions. This problem is inspired by the analogous considerations for the approximate quantum cloning [12]. We completely characterize quantum channels that give the optimal performance (measured in terms of the overlap between the result of the application of the channel to the input state and the desired state) for the approximate creation of superpositions of two unknown vectors characterized by a fixed overlap. Along the way we provide an explicit characterization of an interesting class of covariant  $2 \rightarrow 1$  quantum channels.

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