Using entanglement against noise in quantum metrology [1]

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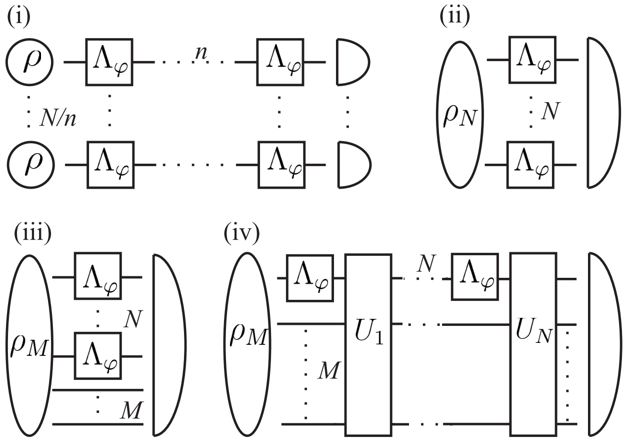

FIG. 1. Quantum metrology strategies. The maps Λ_φ encode the parameter φ to be estimated. (i) sequential scheme: Λ_φ acts n times sequentially on N/n input probes ρ (this is an entanglement-free classical scheme) ; (ii) entangled parallel scheme: an entangled state of N probes ρ_N goes through N maps Λ_φ in parallel; (iii) passive ancilla scheme: the N probes are also entangled with M noiseless ancillas; (iv) active ancilla-assisted scheme: the action of N channels Λ_φ is interspersed with arbitrary unitaries U_i representing interactions of the probe with ancillas. [All the other schemes can be derived from (iv) choosing swap or identity unitaries U_i].

Quantum metrology describes parameter estimation techniques that, by sampling a system N times, achieve precision better than the $1/\sqrt{N}$ scaling of the central limit theorem of classical strategies. Different schemes can beat such limit (Fig. 1): (i) entanglement-free "classical" schemes where N/n independent probes sense the system sequentially thus rescaling the parameter, and hence the error, by n for each probe; (ii) entangled parallel schemes that employ a collective entangled state of the N probes that sample the system in parallel; (iii) passive ancilla schemes, where the N probes may also be entangled with noiseless ancillas; (iv) active ancilla-assisted schemes (comprising all the previous cases) that also encompass all schemes employing feedback: adaptive procedures are described as unitary operations acting on the probes and ancillas between the sensing and the final measurement.

In the noiseless unitary parameter estimation case, $\Lambda_\varphi(\cdot) = U_\varphi \cdot U_\varphi^\dagger$, classical single-probe sequential schemes (i) can attain the same $1/N$ precision as parallel entangled ones (ii) at the expense of an N-times longer sampling time, whereas passive and active ancilla schemes (iii) and (iv) offer no additional advantage [2].

Here, we analyze the performance of these strategies in the presence of specific noise models, and use the results to conjecture a general hierarchy of protocols. Our first result is that in presence of noise (here we analyze dephasing, erasure and damping) entanglement among probes increases the precision over the sequential strategy (i), even thought it fails to do so in the noiseless case, and we provide a quantitative characterization of this advantage proving that the gain in precision is never greater than $\sqrt{e}$. Our second result is to show that (ii) and (iii) are in general asymptotically inequivalent, by demonstrating that (iii) is strictly better than (ii) for amplitude-damping noise. Our third result is to show that the bounds to parallel-entangled strategies (ii) and (iii) derived for a large class of noise models [3, 4] apply asymptotically in N also to the most general strategies (iv), suggesting that active ancilla-based schemes (such a consider for example in error-correction protocols) are not helpful in increasing the precision in the presence of noise.

Finally, we use our results to conjecture a general hierarchy of quantum metrology schemes valid in presence of any uncorrelated noise

$$\begin{array}{lc}
\text{(i)} = \text{(ii)} = \text{(iii)} = \text{(iv)} & \text{decoherence free,} \\
\text{(i)} < \text{(ii)} = \text{(iii)} = \text{(iv)} & \text{dephasing, erasure,} \\
\text{(i)} < \text{(ii)} < \text{(iii)} \overset{?}{=} \text{(iv)} & \text{amplitude-damping,} \\
\textbf{(i)} \overset{?}{\leqslant} \textbf{(ii)} \leqslant \textbf{(iii)} \overset{?}{=} \textbf{(iv)} & \textbf{general conjecture}.
\end{array} \tag{1}$$

Namely, in general, sequential strategies (i) are worse than parallel-entangled ones (ii) [although they are equivalent in the noiseless case], which might in some cases be additionally improved by entangling the probes with noiseless ancillas (iii), but there is no additional asymptotic gain from using active ancilla-aided schemes (iv).

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