From quantum metrological precision bounds to quantum computation speed-up limits

Rafał Demkowicz-Dobrzański[1,*] and Marcin Markiewicz[1,2]

[1] *Faculty of Physics, University of Warsaw, ul. Pasteura 5, PL-02-093 Warszawa, Poland*

[2] *Institute of Theoretical Physics and Astrophysics, University of Gdańsk, ul. Wita Stwosza 57, 80-952 Gdańsk, Poland*

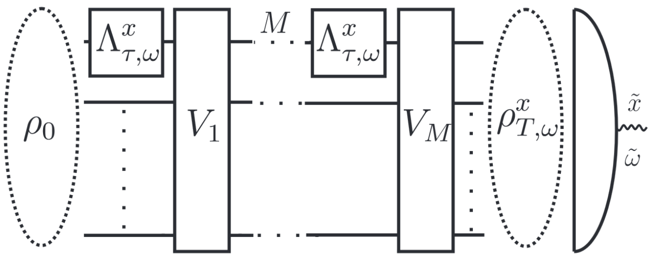

FIG. 1. General scheme for quantum parameter estimation and quantum search problem. Probe state ρ_0 is sent through a sequence of M interrogation steps $\Lambda^x_{\tau,\omega}$ each lasting time τ. Finally it is measured in order to *estimate* frequency parameter ω *knowing* the generating Hamiltonian x (metrology) or *discriminating* from an N element discrete set of generating Hamiltonians x *knowing* the evolution frequency ω (quantum search). Number of steps M is arbitrary but the total interrogation time $T = M\tau$ is a fixed resource.

Quantum metrology as well as quantum computing both aim at exploiting intrinsic quantum features such as coherence and entanglement in order to provide enhancement over performance of corresponding classical protocols. Interestingly, studies of metrological [1, 2] as well as quantum search protocols [3, 4] revealed that in the presence of noise the quadratic performance enhancement is lost in the asymptotic regime of correspondingly large number of probes or large database sizes.

We show that bounds on estimation performance can be directly related to the lower bounds on query complexity of quantum search algorithms by invoking limits on the speed of evolution of quantum states quantified with the help of Quantum Fisher Information (QFI). We recover known lower bounds on noiseless quantum search, whereas in the case of noisy scenarios application of recent powerful quantum metrological methods [5, 6] lead us to a generic conclusion that super-classical scaling of query complexity of search algorithms is asymptotically lost.

Both quantum metrological as well as quantum search tasks are effectively quantum channel discrimination problems (Fig. 1). In order to make use of QFI based metrological precision bounds in deriving speed-up limits of a continuous quantum search in the presence of dephasing consider as in [7, 8] the following average probe distance quantity:

$$\overline{D}_T = \sum_x D(\rho^x_{T,\omega}, \rho_T), \tag{1}$$

where $D(\rho_1, \rho_2)$ is a distance measure to be specified below, $\rho^x_{T,\omega}$ is the final state of the algorithm, whereas ρ_T is the state of the same algorithm (the same set of unitaries V_i), but with the unitary sensing part removed from the oracle queries i.e. $\Lambda^x_{\tau,\omega}$ is replaced by the decoherence map Λ_τ, or equivalently $\omega = 0$ is set. In order to make the connection to metrological bounds, we choose as the distance measure the angular Bures distance:

$$D(\rho_1, \rho_2) = \arccos\left(\mathrm{Tr}\sqrt{\rho_1^{1/2}\rho_2\rho_1^{1/2}}\right), \tag{2}$$

which for infinitesimally close states lying along a trajectory parameterized by a continuous parameter is expressed in terms of the QFI: $D(\rho_\omega, \rho_{\omega+\mathrm{d}\omega}) = \frac{1}{2}\sqrt{F_\omega(\rho_\omega)}\,\mathrm{d}\omega$.

We conclude that for fixed T the quantity $\overline{D}_T$ cannot grow faster that $\sqrt{N}$ and on the other hand for a fixed N it cannot grow faster than $\sqrt{T}$. Unfortunately, without additional technical assumptions on the properties of $\overline{D}_T$, e.g. that asymptotically $\overline{D}_T \propto N^\alpha T^\beta$, we cannot rigorously conclude that

$$\overline{D}_T \leq \sqrt{T}\sqrt{N} \cdot \mathrm{const.} \tag{3}$$

We therefore need to leave the above formula as a natural conjecture that arises from our reasonings, and hope that an approach that would exploit the advantages of both the time and the frequency approaches will be capable of proving the above conjecture rigorously. Using (3) we arrive at the desired result:

$$T \geq \mathrm{const} \cdot N, \tag{4}$$

that the quadratic quantum enhancement in the search algorithm is lost.

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* Rafal.Demkowicz-Dobrzanski@fuw.edu.pl

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