Photon-assisted tunnelling with nonclassical light

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Among the most exciting recent advances in the field of superconducting quantum circuits is the ability to coherently couple microwave photons in low-loss cavities to quantum electronic conductors (e.g. semiconductor quantum dots or carbon nanotubes). These hybrid quantum systems hold great promise for quantum information processing applications; even more strikingly, they enable exploration of completely new physical regimes. Here we study theoretically the new physics emerging when a quantum electronic conductor is exposed to non-classical microwaves (e.g. squeezed states, Fock states) [1]. We study this interplay in the experimentally-relevant situation where a superconducting microwave cavity is coupled to a conductor in the tunneling regime, depicted in Fig. 1.

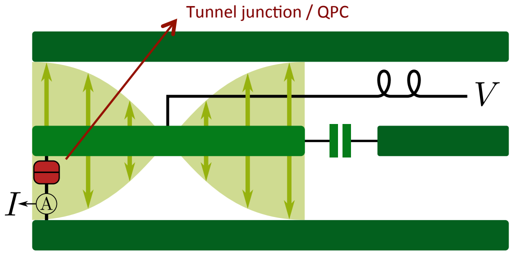

FIG. 1. Schematic showing a resonant mode of a coplanar waveguide resonator with a quantum conductor. The state of the resonant mode provides a quantum ac voltage across the junction.

The physics of a tunnel junction illuminated by a purely classical microwave field is equivalent to simply having an ac bias voltage across the conductor, and the resulting modification of the current is known as photon-assisted tunnelling [2]. Despite the word 'photon' in the effect's name, in this standard formulation there is nothing quantum in the treatment of the applied microwave field. To study a more truly quantum version of photon-assisted tunnelling, one could consider driving a tunnel junction with a quantum microwave field produced in a cavity [3]. If the cavity is not driven, the set-up realises another well-studied quantum transport problem: dynamical Coulomb blockade [4].

Here we develop a comprehensive theory describing how *non-equilibrium*, *driven* states of a microwave cavity influence electronic transport in a coupled tunnel junction, with a particular focus on cavities which are maintained in truly nonclassical states. Generalising both standard photon-assisted tunnelling theory and dynamical Coulomb blockade theory, we show that the emission and absorption of photons by the conductor is naturally characterised by a quasi-probability distribution, which can fail to be positive.

Explicitly, the tunnel current is

$$I(t,V) = e \sum_{\sigma=\pm} \int dE\,\Gamma(\sigma \cdot eV - E) P_{\mathrm{tot}}(E;t,\sigma), \qquad (1)$$

where the function $P_{\mathrm{tot}}(E;t,\sigma)$, describing energy transfer to/from the electromagnetic environment, is given by the causal environmental correlation function evaluated in the absence of tunnelling,

$$G_{\mathrm{env}}(t,\tau;\sigma) = -(i/\hbar)\theta(\tau)\left\langle e^{i\sigma\hat{\phi}(t)} e^{-i\sigma\hat{\phi}(t-\tau)} \right\rangle, \qquad (2)$$

$$P_{\mathrm{tot}}(E;t,\sigma) = -\frac{1}{\pi}\mathrm{Im}\int_{-\infty}^{+\infty} d\tau\, e^{iE\tau/\hbar} G_{\mathrm{env}}(t,\tau;\sigma), \qquad (3)$$

where $\hat{\phi} = (e/\hbar)\int_{-\infty}^{t} \hat{U}(t')dt'$ is the phase operator defined in terms of the Heisenberg-picture environmental voltage operator $\hat{U}(t)$. In terms of time-averaged quantities, this function can be decomposed into a vacuum contribution and an "occupied" contribution,

$$P_{\mathrm{tot}}(E) = \int dE' P_0(E-E') P_{\mathrm{occ}}(E'). \qquad (4)$$

$P_{\mathrm{occ}}(E)$ is the distribution that can fail to be positive.

The resulting negative quasi-probabilities can have a direct influence on both the conductance and finite-frequency current noise of the tunnel junction. We also show that this new quasi-probability distribution has a direct connection to the well-known Glauber-Sudarshan *P*-function of quantum optics. We present results for parameter regimes relevant to state-of-the-art experiments, and show that for sufficiently large tunnel resistances, the tunnel junction acts as a nontrivial and nonlinear probe of the cavity state. Our results suggest the general potential of using quantum conductors as a powerful tool to characterise, and perhaps control, quantum microwave states.

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