Measuring temperature with light

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Introduction.—Temperature is one of the fundamental and arguably one of the most frequently measured physical quantities. Apart from the central role played by the concept of temperature in the fields of thermodynamics and statistical physics, precise temperature measurements are important for many branches of modern science and technology including physics, biology, chemistry, atmospheric sciences, but also material science and microelectronic industry. In this work, we study the classical and quantum limitations that constrain the precision with which we can measure temperature using light.

Classical thermometers.—The precision of classical thermometers is governed by the so-called standard quantum limit $\Delta T \sim c_{SQL}/\sqrt{\bar{N}}$ [1], where c_{SQL} is a constant depending on the properties of the thermometer and the sample, and $\bar{N}$ is the (mean) number of *uncorrelated* particles that make up the thermometer or the time it takes to make a measurement [2]. In our work, we study the most commonly used optical thermometer—the pyrometer. Pyrometers estimate temperature by measuring the flux of thermal radiation emitted by a sample modeled as a black body in thermal equilibrium and using the well-known Stefan-Boltzmann law infer the temperature [3]. We find a very simple and elegant formula that lower bounds the fundamental precision of the *idealized* pyrometer

$$\Delta T \geq \sqrt{\frac{k_B}{4\sigma S \delta t T}} \qquad \text{with} \qquad \sigma = \frac{\pi^2 k_B^4}{60\hbar^3 c^2}, \tag{1}$$

where σ is the Stefan-Boltzmann constant, S is the surface area of the sample we are probing, δt is the response time of the device, that is, the time it takes to make a measurement, and T is the local temperature of the sample. This result has a SQL-type scaling with respect to δt and we also show that this scaling is optimal. Assuming $S = 1\text{cm}^2$ and $\delta t = 10\text{ms}$, we find that the local precision of such a measurement near $T = 298\text{K}$ is $\Delta T = 452n\text{K}$. Therefore, at the fundamental level pyrometers are highly precise temperature sensors.

Quantum interferometric thermometers.—Recent years have witnessed a growing interest in applying various ideas, taking advantage of the quantum-mechanical features of nature, to the problem of temperature measurements. Specifically, in Ref. [2], it was shown that using the interferometric techniques of phase estimation theory the classical precision in temperature estimation may be improved to the so-called Heisenberg limit $\Delta T \sim c_{HL}/\bar{N}$ [1], where c_{HL} is of the order of unity and $\bar{N}$ is the (mean) number of *correlated* particles that make up the thermometer. Unfortunately, this improvement occurs only for measurements in the absence of decoherence; typically, the presence of noise reduces the Heisenberg-limited precision back to the classical SQL-type scaling $\Delta T \sim c/\sqrt{\bar{N}}$, where $c < c_{SQL}$ [4]. In our work, we determine the fundamental precision of a realistic optical interferometric thermometer. To this end, we provided a detailed description of such a device by studying the interaction of the optical probe system prepared in a single-mode Gaussian state with a heated sample modeled as a thermal dissipative reservoir [5]. By assuming a linear dependence of the optical phase shift on the temperature of the sample, that is, assuming $\varphi = \alpha T$, and using the standard methods of estimation theory we find that the single-mode squeezed-vacuum state is optimal for temperature estimation with the asymptotic error given by

$$\Delta T \geq \frac{1}{\alpha}\sqrt{\frac{(1-\eta)(1+2N)}{4\eta \bar{N}}}, \tag{2}$$

where η denotes photon loss, N is the mean number of thermal photons present in the sample, and $\bar{N}$ is the mean number of photons present in the probe. We compare this precision limit against the theoretical limit of the idealized pyrometer. This comparison is provided for one of the most important classes of materials used in many quantum optics experiments, namely, for the non-linear crystals. As we find the best precision achieved by our interferometric approach is $\Delta T \approx 1.4n\text{K}$. Therefore, the quantum interferometric thermometer provides a superior performance in temperature sensing even when compared with the idealized pyrometer. One of the main advantages of interferometric thermometry is its noninvasiveness as highly precise temperature measurements can be obtained without disturbing sample's temperature. We predict that quantum interferometric thermometers will prove useful for ultra-precise temperature sensing and stabilization of quantum optics experiments based on the non-linear crystals and atomic vapors.

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