State Reconstruction of an Oscillator Network in an Optomechanical Setting

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The scheme developed in this work allows the full quantum state of a network of harmonically interacting mechanical oscillators to be reconstructed. This is accomplished by coupling one distinguished oscillator of the network via radiation pressure to the optical mode of a cavity. This miinimal requirement of only one coupled oscillator renders the scheme relatively noninvasive of the mechanical state.

The Hamiltonian for the system, after linearisation of the coupling, may be written:

$$H = H_0 + H_{\text{int}} \tag{1}$$
$$H_0 = \sum_n \omega_n b_n^\dagger b_n + \sum_{n<m} J_{nm}(b_n b_m^\dagger + b_n^\dagger b_m) \tag{2}$$
$$+ \sum_{n<m} K_{nm}(b_n b_m + b_n^\dagger b_m^\dagger) \tag{3}$$
$$H_{\text{int}} = g(t)X(b_1 + b_1^\dagger) \tag{4}$$

where b_n are the mechanical modes, ω_n are the mechanical frequencies, J_{nm} and K_{nm} are the coupling constants for the network, g(t) is a time-dependent optomechanical coupling strength, $X = a + a^\dagger$ is the optical mode's position quadrature and the oscillator labelled 1 is the distinguished one coupled to the optical mode.

Define S as the symplectic matrix that brings the network into the basis of normal modes:

$$S = \begin{pmatrix} S_1 & S_2 \\ S_2^* & S_1^* \end{pmatrix}. \tag{5}$$

The oscillator network may be transformed into the basis of normal modes, revealing

$$H_0 = \sum_n \nu_n d_n^\dagger d_n \tag{6}$$
$$H_{\text{int}} = g(t)X \sum_n G_n d_n + G_n^* d_n^\dagger \tag{7}$$

where $G_n = (S_1 - S_2)^*_{n1}$, d_n are the normal modes of the network and ν_n its eigenfrequencies. Considering an interaction picture defined by H_0, we have

$$H_I = g(t)X \sum_j h_j(t) \tag{8}$$

where $h_j = G_j d_j e^{-i\nu_j t} + G_j^* d_j^\dagger e^{i\nu_j t}$. The dynamics of the system is solved by

$$U = e^{i\Psi X^2} D(X\beta) \tag{9}$$

where $\Psi = \sum_j \psi_j$, $\beta = \begin{pmatrix}\beta_1 & \beta_2 & \dots & \beta_N\end{pmatrix}^\top$ and

$$\beta_j = -iG_j^* \int_0^t g(s) e^{i\nu_j s} ds \tag{10}$$
$$\psi = -\int_0^t \text{Im}(\beta_j \dot{\beta}_j^*) ds \tag{11}$$

with N the number of oscillators.

Define a measurement operator $L = -4\Psi X + P$ (with P the momentum conjugate with X) and the mechanical mode quadratures by $Q_{\theta_j} = d_j e^{-i\theta_j} + d_j^\dagger e^{i\theta_j}$ [where $\theta_j = \arg(\beta_j) + \frac{\pi}{2}$]. Note that these quadratures are defined in terms of the normal mode operators. Given an initial separable state $\rho = |0\rangle\langle 0| \otimes \rho_0$ with $|0\rangle$ the optical vacuum and ρ_0 an arbitrary state of the network, after an appropriate evolution a measurement of the light is made. A histogram of such measurement results produces a set of statistical moments which are related to the moments of the quadratures Q_{θ_j} via the following relation:

$$\langle L^n \rangle = \sum_{k_0+k_1+\dots k_N = n} \binom{n}{k_0, k_1, \dots k_N} \langle 0|P^{k_0}|0\rangle$$
$$\langle \Pi 1 \le j \le N(-2|\beta_j|Q_{\theta_j})^{k_j}\rangle \tag{12}$$

Inverting the relation above it is possible to obtain a suitable collection of moments for a particular quadrature. The latter can be selected by changing the time profile of the interaction strength $g(t)$. Thus a suitable set of marginals can be extracted from which, in turn, the phase space distribution of the network can be reconstructed using standard tomographic techniques.