Localization versus Gaussianity in the measurement space

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General features of N-qubit systems can be described by discrete distributions in the space of symmetric measurements [1], obtained by projecting an appropriately defined discrete Husimi $Q(\alpha,\beta)$-function (α,β are N-component binary strings) from $2^N\times 2^N$ phase-space into 3-dim space of permutation invariants $m=h(\alpha), n=h(\beta), k=h(\alpha+\beta)$, where $h(\kappa)=\sum_{i=1}^{N} k_j$, $\kappa=(k_1,...,k_N)$, $k_j\in\mathbb{Z}_2$. Such distributions, $\tilde{Q}(m,n,k)$, $0\leq m,n,k\leq N$, contain the full and non-redundant information about results of measurements of any collective (invariant under particle permutations) observable in an arbitrary (not necessarily symmetric state).

The $\tilde{Q}$-function are especially useful both for visualization purposes and for the analysis of general properties of quantum states from measured data for large particle systems. In the macroscopic limit, $N\gg 1$, the $\tilde{Q}$-function tends to a smooth distribution and is *bounded* by a Gaussian function

$$\tilde{Q}(\mathbf{x})\subset G(\mathbf{x})\sim\exp\left(-N(\mathbf{x}-\mathbf{x}_0)\mathbf{T}^{-1}(\mathbf{x}-\mathbf{x}_0)\right), \qquad (1)$$

where $\mathbf{x}=(m,n,k)/N$, $\mathbf{x}_0\sim\langle\mathbf{S}\rangle$, $S_j=\oplus\sum_{i=1}^{N}\sigma_j^{(i)}, j=x,y,z$, are the collective operators and the dispersion matrix $\mathbf{T}$ is related to the covariance matrix $\mathbf{\Gamma}_{ij}=\langle\{S_i,S_j\}\rangle-\langle S_i\rangle\langle S_j\rangle$ and average values $\langle\mathbf{S}\rangle$.

The notion of localization and Gaussianity in the measurement space can be introduced:

States are localized if the width of the $\tilde{Q}$-function is much less than the extension of the measurement space. The localized states are characterized by the absence of long-range quantum correlations, $\lim_{N\to\infty}M_p/N^p=0$, where M_p is the p-th order central moment of the observable $\mathbf{S}\cdot\mathbf{n}$ ($\mathbf{n}$ is a unit vector), and can be roughly characterized by the volume of the envelop $G(\mathbf{x})$. For pure states the localization property is related to the concept of the macroscopic quantumness [2].

A state is of Gaussian type if it is well described by the envelop (1): $\tilde{Q}(\mathbf{x})\sim G(\mathbf{x})$, so that the outcomes of the symmetric measurements, i.e. average values $\langle(\mathbf{S}\cdot\mathbf{n})^p\rangle$, $p\ll N$, depend on a small number of parameters, which are essentially the lowest order moments of the collective variables $\mathbf{S}$. The Gaussianity can be quantified in terms of the Kullback divergence,

$$D_{Kl}\sim\int d\mathbf{x}\tilde{Q}(\mathbf{x})\ln\tilde{Q}(\mathbf{x})/G(\mathbf{x}). \qquad (2)$$

We provide a comparative analysis of several (W-states, GHZ and locally transformed GHZ, singlet-like states, discrete coherent states, domain wall states, etc.) pure states and the corresponding incoherent mixtures in the limit $N\gg 1$ (see Fig.1). The distributions characterized by large values of the volume of the envelope (V_A) and the Kullback divergence (D_{Kl}) correspond to non-localized and non-Gaussian states.

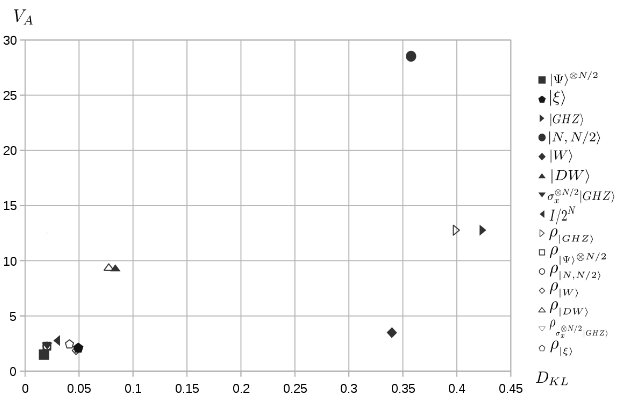

FIG. 1. Volume of the envelop V_G (1) $\sim\sqrt{\det\mathbf{T}}$, N=100, versus the Kullback divergence (2) N=10. Large values of the volume correspond to non-localized states; large values of the Kullback entropy D_{Kl} correspond to non-Gaussian states. Here $|\xi\rangle^{\otimes N}$ is a direct product of single qubit states with $\mathbf{n}=(1,1,1)/\sqrt{3}$; $|N,N/2\rangle$ is the Dicke state with a half number of excitations; $|DW\rangle\sim|000...\rangle+|100...\rangle+...+|111...\rangle$ is the domain wall state; $\sigma_x^{\otimes N/2}|GHZ\rangle$ is the GHZ-state with $N/2$ flipped qubits; $|\Psi\rangle^{\otimes N/2}$ is N qubit the singlet state: a product of $N/2$ anti-symmetric Bell states; mixed states corresponding to pure states $|\psi\rangle$ are denoted as $\varrho_{|\psi\rangle}$

We show that:

a) Localized states (with $\lim_{N\to\infty}Tr\mathbf{T}/N=0$) can reflect essentially non-Gaussian behavior;

b) States with long-range quantum correlations (non-localized) can show a high-degree Gaussianity;

c) Incoherent mixtures corresponding to pure localized states are Gaussian.

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[1] A.B. Klimov, et al. Phys. Rev.A **70**, 062101 (2014).
[2] F. Frowis, et al New J.Phys **14** 093039 (2012).