Degenerate parametric oscillation in membrane cavity optomechanics

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The last decades have seen the birth of a plethora of new technologies working in the quantum regime, starting with the laser, and including nonlinear optics, trapped ions and atoms, cavity quantum electrodynamics, or, more recently, superconducting circuits and optomechanical resonators. Apart from their potential for quantum computation, simulation, metrology, and communications, these systems have allowed us to reach physical scenarios that were nothing but a dream (or a 'gedanken' experiment) for the founding fathers of quantum mechanics.

We keep deepening into the possibility of using new technologies to access phenomena that, though predicted and theoretically analyzed long ago, have eluded observation so far. In particular, we show that modern *optomechanical setups* allow for the implementation of *degenerate parametric oscillation*, one of the best studied models in the field of open systems dynamics (together with lasing), holding the paradigm of a dissipative phase transition whose associated spontaneously broken symmetry is discrete.

The main motivation for studying such a model came from nonlinear optics, in particular from the possibility of implementing it in optical parametric oscillators [1]: when a cavity containing a second order nonlinear crystal is pumped with a laser at frequency ω_p, the parametric down-conversion phenomenon occurring in the crystal should be in principle able to generate a field at the subharmonic frequency $\omega_p/2$, while the quadratic nonlinearity of the system allows this down-converted field to have either a phase 0 or π with respect to the pumping laser. This way, a discrete symmetry is broken once the down-converted field starts oscillating inside the cavity, what happens only if the pumping power exceeds some threshold value above which the nonlinear gain can compensate the losses through the mirror (phase transition). Defining the annihilation operator a for photons in the subharmonic mode, the model is formulated as the following evolution equation for the state ρ of the subharmonic field:

$$\dot{\rho} = [\sigma(a^{\dagger 2} - a^2), \rho] + \gamma \mathcal{D}_a[\rho] + \Gamma \mathcal{D}_{a^2}[\rho], \qquad (1)$$

with dissipators $\mathcal{D}_b[\rho] = 2b\rho b^\dagger - b^\dagger b\rho - \rho b^\dagger b$; the first and last terms describe, respectively, coherent exchange and irreversible loss of photon pairs into the pumping laser, while the second one accounts for damping through the mirrors, trying to bring the mode to its vacuum state.

Unfortunately, the $\omega_p/2$ degenerate down-conversion has to compete with non-degenerate processes in which the photon pairs are generated at cavity frequencies ω_1 and ω_2 such that $\omega_p = \omega_1 + \omega_2$ (energy conservation), and it is simple to show that phase-matching (momentum conservation) always gives preference to one of these processes. In other words, despite the great deal of work invested on this system, above-threshold continuous-wave degenerate optical parametric oscillators do not exist in reality.

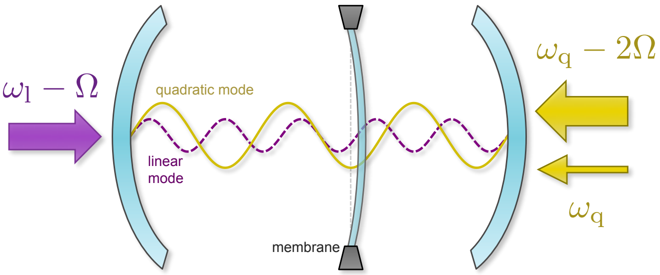

FIG. 1. Setup proposed for the observation of degenerate parametric oscillation in a mechanical degree of freedom. The membrane is dispersively coupled to two driven cavity modes. One of them, denoted by 'linear' because its frequency depends linearly on the membrane's displacement (ω_l is its bare value), tries to cool down the membrane's fundamental mode (with oscillation frequency Ω) close to its quantum mechanical ground state, hence controlling the rate γ in Eq. (1). The other mode, denoted by 'quadratic' because in this case its frequency varies quadratically with the membrane's displacement, contains two tones, one at the resonant frequency ω_q and another at the two-phonon sideband $\omega_q - 2\Omega$. The combination of these two tones induces the coherent and incoherent two-phonon processes needed for degenerate parametric oscillation, and, through their power, they allow for an independent tuning of Γ and σ in Eq. (1).

In this work we prove that modern quantum technologies in which a mechanical degree of freedom is dispersively coupled to the light field of a cavity, allow for a realistic implementation of this long-awaited model. We focus on a system consisting of an oscillating dielectric membrane embedded in an optical cavity, see Fig. 1, and show how the conditions under which these setups are currently operated [2], together with the proper multi-chromatic driving, make the experimental analysis of degenerate parametric oscillation accessible with present technology.

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