True precision limits in quantum metrology

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Quantum mechanics provides insight into fundamental limits on the achievable measurement precision that cannot be beaten irrespectively of the extent of any improvements in measurement technology. The best known example is that of the optical phase measurement where difference of phase delays φ in the arms of interferometer in the absence of decoherence can only be measured up to a precision known as Heisenberg limit that scales as $\Delta\varphi \geq 1/N$ where N is the number of photons sent into the setup. Presence of decoherence typically prevents from reaching the Heisenberg scaling, and it may be demonstrated that for the generic uncorrelated noise processes classically scaling bounds $\Delta\varphi \geq \mathrm{const}/\sqrt{N}$ hold, limiting quantum enhancement to a constant factor precision improvement [1].

Most of the bounds derived in the field of quantum metrology, including the ones mentioned above, are applications of the Quantum Cramér-Rao (C-R) bound [2] based on calculation of the quantum Fisher information (QFI). It is known that in principle C-R bound may be saturated by some particular measurement and maximum likelihood estimator in the limit of number of repetitions of experiment k going to infinity.

Practical implications of this last statement are far form obvious, however. The QFI depends only on the local properties of the state at a given parameter value φ. Saturating the C-R bound may therefore require unrealistically good prior knowledge on the value of the estimated parameter. Moreover, in order to quantify the performance in terms of the *total* resources consumed, i.e. kN, one needs to know the behavior of the required number of repetitions k with the increase of N, which can be highly nontrivial.

However, there are also alternative ways of deriving bounds on the precision, that does not suffer from the above mentioned deficiencies. In particular in the Bayesian approach one explicitly takes into account the prior knowledge about the parameter value, represented by a probability distribution $p(\varphi)$. Finding the minimal average Bayesian error is much more demanding than maximization of QFI over input states, yet, once the solution is found it yields an explicit estimation procedure that saturates the bound.

In [3] we have shown in which situations Bayesian error is asymptotically equal to the C-R bound. This allowed us to prove that in the presence of local uncorrelated decoherence optimal C-R bound is always asymptotically saturated by a Bayesian procedure. On the other hand, in the decoherence free case we were able to show that *irrespectively* of prior knowledge, minimal Bayesian error always converges to π/N which is worse than conventional Heisenberg scaling $1/N$ obtained from C-R bound by a factor of π (see fig. (1)). This shows that, contrary to the intuition,

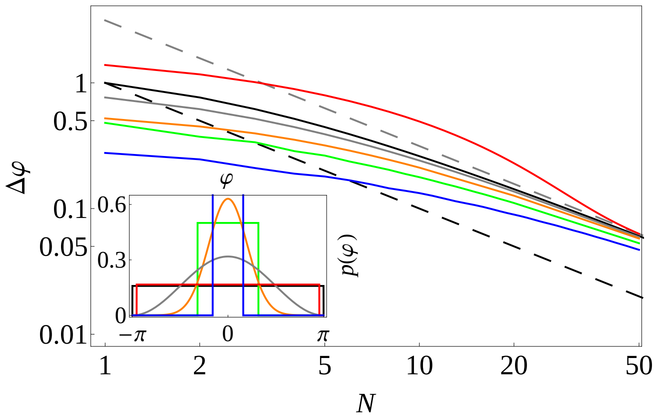

FIG. 1. Bayesian cost for various prior distributions $p(\varphi)$ converge to π/N (gray dashed), not to $1/N$ C-R bound (black dashed). The shapes of prior distributions are depicted on the inset.

even with very large prior knowledge C-R bound cannot be saturated asymptotically in one shot by any measurement scheme. We considered also the case of collective dephasing which is an example of global decoherence channel and cannot be decomposed into local uncorrelated channels. In such case we obtained that there is no connection between C-R bound and Bayesian error, moreover, the last quantity depends on prior knowledge.

Our work confirm that in the presence of uncorrelated decoherence the asymptotic limits on precision of quantum metrological schemes may be credibly calculated using the C-R bound based approach whereas in the decoherence-free unitary parameter estimation a π factor correction needs to be included irrespectively of the extent of prior knowledge. These observations provide a firm ground for the use of the QFI as a sensible figure of merit in analyzing the performance of quantum enhanced metrological protocols based on definite-particle number states.

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