Quantum cooling and squeezing of a levitating nanosphere via time-continuous measurements

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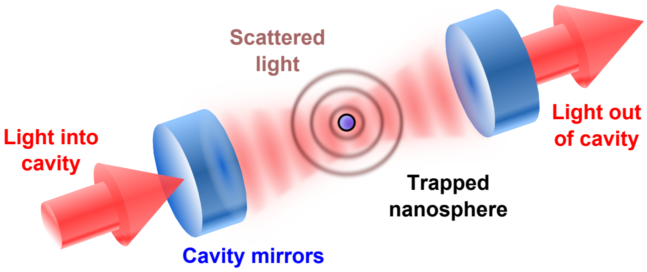

FIG. 1. Schematic of the experimental set-up for the quantum control of a levitated dielectric nanosphere within an optical cavity.

With the purpose of controlling the steady state of a dielectric nanosphere levitated within an optical cavity we study its conditional dynamics under simultaneous sideband cooling and additional time-continuous measurement of either the output cavity mode or the nanosphere's position. We consider two quantum degrees of freedom; the cavity electromagnetic mode and the mechanical motion of a trapped nanosphere, described respectively by bosonic operators a and b satisfying the commutation relations $[a, a^\dagger] = [b, b^\dagger] = \mathbb{1}$. By assuming the cavity driven by a laser at frequency ω_L, the Hamiltonian describing the interaction between the two modes reads [1]

$$H = \omega_m b^\dagger b - \Delta a^\dagger a + g(a + a^\dagger)(b + b^\dagger), \qquad (1)$$

where ω_m is the mechanical frequency and we have already transformed the Hamiltonian to a frame rotating at the frequency ω_L, such that $\Delta = \omega_L - \omega_c$ denotes the detuning from the cavity resonance ω_c.
The corresponding conditional evolution due to general-dyne time-continuous measurement on both the cavity mode and the oscillator is described by the following stochastic master equation [1, 2]

$$d\varrho = -i[H, \varrho]\, dt + \kappa\, \mathcal{D}[a]\varrho\, dt + \Gamma\, \mathcal{D}[b + b^\dagger]\varrho\, dt + \qquad (2)$$
$$+ \sqrt{\eta_1 \kappa}\, \mathcal{H}[a e^{i\phi}]\varrho\, dw_1 + \sqrt{\eta_2 \Gamma}\, \mathcal{H}[b + b^\dagger]\varrho\, dw_2\,, \qquad (3)$$

where $\mathcal{D}[O]\varrho = O\varrho O^\dagger - (O^\dagger O \varrho + \varrho O^\dagger O)/2$, $\mathcal{H}[O]\varrho = O\varrho + \varrho O^\dagger - \mathrm{Tr}[(O + O^\dagger)\varrho]\varrho$ and dw's are uncorrelated Wiener increments, such that $dw_j dw_k = dt\, \delta_{jk}$. The term $\sqrt{\eta_1 \kappa}\mathcal{H}[a e^{i\phi}]$ describes the effect of continuous homodyne on the output cavity mode with efficiency η_1, where the phase ϕ can be adjusted by choosing the optical phase of the monitored quadrature operator. Analogously, the term $\sqrt{\eta_2 \Gamma}\, \mathcal{H}[b + b^\dagger]$ describes the effect of continuous monitoring of the oscillator position, with efficiency η_2.
We will discuss the efficiency of our protocols by considering the effect on the mechanical oscillator steady-state properties, comparing in particular the results obtainable by sideband cooling only and by combining sideband cooling with time-continuous measurements. Our results can be summarized as follows:

- time-continuous measurements of either the cavity mode or the oscillator position, accompanied by Markovian feedback, are able to stabilize the nanosphere motion for all the values of the detuning Δ and of the measurements efficiencies.

- The addition of time-continuous homodyne monitoring of the cavity output plus Markovian feedback greatly improves the performance that one would have obtained with sideband cooling only. For the realistic values of physical parameters considered in our study, while sideband cooling would prepare a phase-insensitive steady-state characterized by $n_{\mathrm{ph}} \gtrsim 30$ phonons on average, the addition of continuous homodyne measurements of the cavity output would prepare a squeezed steady-state, with $n_{\mathrm{ph}} < 10$ phonons. In particular, this is true for a large range of detuning values, which relaxes the requirements of sideband resolution to cool down the oscillator. Notice that the results are further improved if a simultaneous measurement of the oscillator position is carried out.

- If we consider the state-of-the-art experimental setup described in [3] where the dielectric nanosphere is trapped by the field of a high-finesse cavity, the proposed measurement protocols are in principle able to prepare a quantum squeezed state with less than $n_{\mathrm{ph}} = 1$ phonons at steady-state.

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