Priority choice two-qubit tomography

Karol Bartkiewicz,[1,2,*] Antonín Černoch,[3] Karel Lemr,[2] and Adam Miranowicz[1]

[1]*Faculty of Physics, Adam Mickiewicz University, PL-61-614 Poznań, Poland*
[2]*RCPTM, Joint Laboratory of Optics of Palacký University and Institute of Physics of Academy of Sciences of the Czech Republic, 17. listopadu 12, 772 07 Olomouc, Czech Republic*
[3]*Institute of Physics of Academy of Science of the Czech Republic, Joint Laboratory of Optics of Palacký University and Institute of Physics of Academy of Sciences of the Czech Republic, 17. listopadu 50A, 77207 Olomouc, Czech Republic*

Two-qubit density matrix tomography is a key issue for developing quantum-enhanced technologies. Two-qubit polarization states are particularly important in this context as they include, e.g., two polarization-entangled photons, an essential element of quantum communication. In this talk we report on our results on an experimental comparison of three popular tomographic protocols and a recently proposed optimal protocol [1] for a pair of polarization qubits. These tomographic protocols are mathematically equivalent to solving the $Ax = b$ linear system problem, where A is the *coefficient matrix*, b is the *observation vector* that contains the measured data, and $x = (x_1, ..., x_{16})$ is a real vector describing the unknown two-qubit state $\rho(x)$. The state in question, reconstructed by solving the linear system problem, is always perturbed because of the unavoidable noise δb involved in observations $[A(x + \delta x) = b + \delta b]$. Thus, the quality of the reconstructed state can be improved only by choosing A associated with the smallest condition number $\kappa(A)$ [1–3]. The significance of the condition number can be understood through the following inequality [2]:

$$\frac{1}{\kappa(A)} \frac{||\delta b||}{||b||} \leq \frac{||\delta x||}{||x||} \leq \kappa(A) \frac{||\delta b||}{||b||}. \quad (1)$$

If a condition number $\kappa(A)$ (defined here by spectral norm) is equal (or very close) to one, then small relative perturbations in the observation vector b imply equally small relative perturbations in the reconstructed vector x. As shown in Ref. [1], optimal tomography provides $\kappa(A) = 1$, tomography proposed by James *et al.* [4] is described by $\kappa(A) = 60.1$, standard 36 state tomography [5, 6] gives $\kappa(A) = 3$, and that based on mutually-unbiased bases [7] yields $\kappa(A) = 5$. We demonstrate that the states reconstructed with these different protocols are less perturbed if the condition number is small, which follows from inequality (1). Using inequality (1) we demonstrate how to estimate trace distance E between the reconstructed and unperturbed states for real measured data (see Fig. 1). We conclude that our optimal tomography offers the most robust solution.

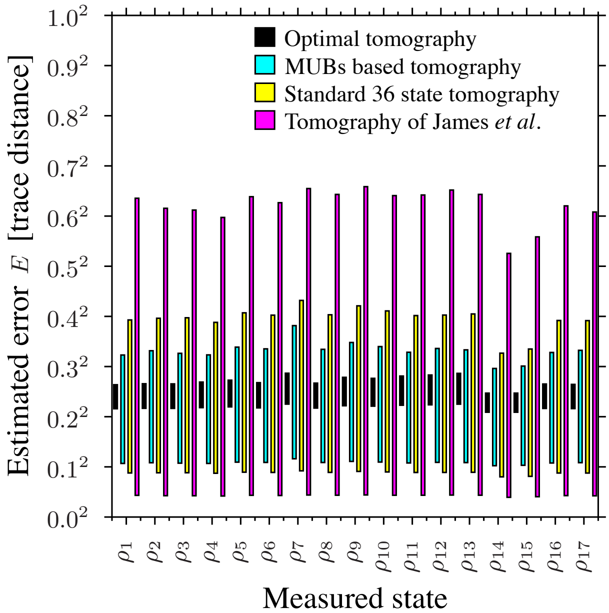

FIG. 1. The presented bars contain the most likely errors E (trace distance from the ideally reconstructed state) in state estimation for the chosen 17 states (including separable, maximally entangled and partially entangled states) which were experimentally reconstructed with the four protocols. The errors can be more precisely estimated if the condition number is small. The maximum error can be estimated as twice the upper bound of the relevant bar.

* bark@amu.edu.pl

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