A distance-type measure of Gaussian EPR-steering

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In quantum mechanics, one of the salient features of the entangled bipartite states is their nonlocality, as pointed out by Einstein, Podolsky, and Rosen (EPR) eighty years ago [1]. A local measurement performed on one party can change the state of the other (remote) one, provided that strong quantum correlations exist between parties. Schrödinger called steering this consequence of nonlocality [2]. A significant piece of progress in classifying the quantum correlations associated with non-separability, EPR-steering and Bell non-locality was the careful analysis carried out by Wiseman, Jones and Doherty [3, 4]. Besides some interesting examples of steerability conditions found for qubit states with high symmetry, an important result emerged in Refs.[3, 4] for the emblematic continuous-variable states described by Gaussian density operators. As it is well known, the Gaussian states of the quantum radiation field are important resources in many quantum information protocols. They are experimentally quite accesible and have the virtue that many calculations can be carried out analytically. In Refs.[3, 4], a necessary and sufficient condition of non-EPR-steerability for two-party multimode Gaussian states could be established by restricting the set of local measurements to the Gaussian ones.

In the present work we examine closely the EPR-steerability of an especially interesting mixed Gaussian state prepared by the action of a two-mode squeeze operator on a particular product state

$$\rho_{\bar{n}} = \rho_T(\bar{n}) \otimes |0\rangle\langle 0|, \tag{1}$$

where $\rho_T(\bar{n})$ describes a one-mode thermal state with mean photon occupancy $\bar{n}$. What we get by this action is a squeezed thermal state (STS) having the property

$$\det(\mathcal{V} + \frac{i}{2}J) = 0, \tag{2}$$

where J is the standard matrix of the symplectic form on $\mathbb{R}^4$. Equation (2) assures the non-separability of the given STS for any $\bar{n}$ [5]. We here apply the non-EPR-steerability condition [3, 4] in the form put forward in Ref.[6] which is remarkably formulated in terms of the global and local purities. Non-EPR-steerability from mode 1 to mode 2 simply means

$$\det(\mathcal{V}) \geq \frac{1}{4}\det(\mathcal{V}_1). \tag{3}$$

It turns out that the given states are steerable from mode 1 to mode 2 for any $\bar{n}$. On the contrary, EPR-steerability from mode 2 to mode 1 requires that the squeeze parameter exceeds a threshold dependent of mixing

$$\sinh(r) > \sqrt{\frac{\bar{n}}{\bar{n}+1}}. \tag{4}$$

When the condition (4) is met the here studied STSs are EPR-steerable in both ways. The principal result of our work is the evaluation of the maximal fidelity between a given EPR-steerable state and any non-steerable Gaussian state for both ways of local action. Its evaluation was performed using Uhlmann fidelity between two-mode Gaussian states written in Ref.[7]. This gives us the possibility to define a degree of EPR-steerability in both cases based on Bures metric after the pattern first described in our paper [5]. We found that the maximal fidelity is independent of the degree of mixing and steering does not exceed entanglement in both cases.

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