Interconversion of pure Gaussian states requiring non-Gaussian operations

Michael G. Jabbour,[1, *] Raúl García-Patrón,[1] and Nicolas J. Cerf[1]

[1]*Quantum Information and Communication, Ecole Polytechnique de Bruxelles, CP 165, Université libre de Bruxelles,*
1050 Bruxelles, Belgium

Quantum entanglement plays a major role in quantum computation and information theory, where it is a key resource enabling a vast variety of tasks, as well as a central component of the security analyses of quantum key distribution. A fundamental problem in the theory of quantum entanglement consists in classifying entangled states into different equivalence classes and studying the possibility (or impossibility) of transforming entangled states between each other. In the usual scenario, one considers interconversions between different sets of states that rely on local operations supplemented with classical communication (LOCC).

In this work [1], we envisage the interconversion between states of the electromagnetic field, and focus in particular on the set of pure Gaussian states, which are of great significance in quantum optics and continuous-variable quantum information theory [2]. The interconversion between Gaussian states has been studied in a few earlier works, but many problems remain unsolved today. In particular, most works have focused on the entanglement transformations of Gaussian states using Gaussian processes only. Notably, the work by Giedke *et al.* [3] provides a necessary and sufficient condition for the interconversion between pure Gaussian states when restricting to Gaussian local operations with classical communication (GLOCC).

The question that we investigate in the present work is whether it is possible to achieve, using a non-Gaussian LOCC, transformations between Gaussian states that are otherwise inaccessible by a GLOCC, i.e., do not satisfy Giedke *et al.*'s condition. We develop a systematic approach to explore the possible interconversions between Gaussian states that are not accessible by GLOCC. We broaden the analysis by providing a sufficient condition for the existence of a LOCC transformation between pure bipartite $N \times N$ Gaussian states that generalizes the criterion proved by Giedke *et al.*, at the price of loosing its necessary character. To achieve this goal, we use the theory of majorization, which provides an ideal tool to investigate the conditions for interconverting pure bipartite states using LOCC transformations [4]. Specifically, a bipartite pure state $|\psi\rangle$ can be transformed via a deterministic LOCC into $|\psi'\rangle$ if and only if there exists a bistochastic (column-stochastic in the infinite dimensional case) matrix $\mathbf{D}$ that maps the vectors of eigenvalues $\boldsymbol{\lambda'}$ of $|\psi'\rangle$'s reduced state onto the vectors of eigenvalues $\boldsymbol{\lambda}$ of $|\psi'\rangle$'s reduced state, i.e., $\boldsymbol{\lambda} = \mathbf{D}\boldsymbol{\lambda'}$. The technical novelty of our work consists in finding a systematic way of constructing such matrices $\mathbf{D}$. This is possible using quantum-limited amplifiers $\mathcal{A}$ and pure-loss channels $\mathcal{L}$, whose tensor products act on some specific tensor prod-
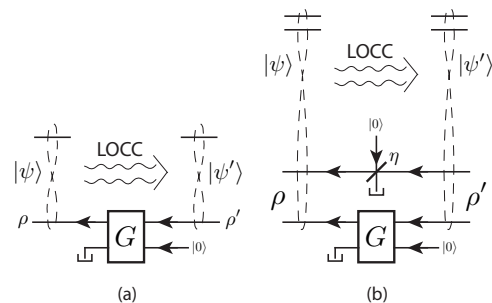

FIG. 1. A bipartite pure state $|\psi\rangle$ can be transformed into $|\psi'\rangle$ with a LOCC if and only if the vector of eigenvalues $\boldsymbol{\lambda'}$ of ρ' can be mapped onto the vector of eigenvalues $\boldsymbol{\lambda}$ of ρ with a column-stochastic matrix $\mathbf{D}$. (a) case of 1×1 modes, where matrix $\mathbf{D}^{\mathcal{A}}$ acts on the vector of eigenvalues of Fock-diagonal states similarly as a quantum-limited amplifier $\mathcal{A}$ of gain G acts on Fock-diagonal states ($\mathbf{D}^{\mathcal{A}}$ is column-stochastic provided that $G \geq 1$). (b) case of 2×2 modes, where matrix $\mathbf{D}^{\mathcal{L}} \otimes \mathbf{D}^{\mathcal{A}}$ acts on the vector of eigenvalues of tensor product of Fock-diagonal states similarly as the tensor product of a pure loss channel $\mathcal{L}$ of transmittance η with a quantum-limited amplifier $\mathcal{A}$ of gain G acts on tensor products of Fock-diagonal states ($\mathbf{D}^{\mathcal{L}} \otimes \mathbf{D}^{\mathcal{A}}$ is column-stochastic provided that $\eta G \geq 1$).

ucts of thermal states in such a way that the vector of eigenvalues of the state evolves according to a column-stochastic matrix $\mathbf{D}$, as it is shown on Fig. 1.

While our technique is applicable to an arbitrary number of modes for each party, it allows us to exhibit surprisingly simple examples of 2×2 Gaussian states that necessarily require non-Gaussian local operations to be transformed into each other.

---

* mjabbour@ulb.ac.be

[1] M. G. Jabbour, R. García-Patrón and N. J. Cerf, Phys. Rev. A **91,** 012316 (2015).

[2] C. Weedbrook, S. Pirandola, R. García-Patrón, T. Ralph, N. J. Cerf, J. H. Shapiro, and S. Lloyd, Rev. Mod. Phys. **84,** 621 (2012).

[3] G. Giedke, J. Eisert, J. I. Cirac and M. B. Plenio, Quant. Inf. Comp. **3,** 211 (2003).

[4] M. A. Nielsen, Phys. Rev. Lett. **83,** 436 (1999).