A new quantum scheme for normal form games

Piotr Frąckiewicz[1,*]

[1] *Institute of Mathematics, Pomeranian University*
ul. Arciszewskiego 22d, 76-200 Słupsk, Poland

A fifteen year period of research on quantum games results in many ideas of how a quantum game might look like and how it might be played. Certainly, the quantum scheme for 2×2 games introduced in [1] (the EWL scheme) has become one of the most common models and it has already found application in more complex games (for example, [2]). However, the more complex a classical game is the more sophisticated techniques are required to find optimal players' strategies in the EWL-type scheme. While in the scheme for 2×2 games the result of the game depends on six real parameters (each players' strategy is a unitary operator from SU(2), and it is defined by three real parameters), the EWL-type scheme for 3×3 games would already require 16 parameters to take into account [3], [4]. One way to avoid cumbersome calculations when studying a game in the quantum domain was presented in [5]. The authors defined a model (the MW scheme) for quantum game where the players' unitary strategies were restricted to the identity and bit-flip operator. Then, the game became *quantum* if the players' local operators were performed on some fixed entangled state $|\Psi\rangle$ (called the players' joint strategy). The MW scheme appears to be much simpler than the EWL scheme. The number of pure strategies of each player is the same as in the classical game. Thus, the complexity of finding a rational solution is similar in both a classical game and the corresponding quantum counterpart. Unfortunately, that simple scheme exhibits some undesirable properties. First, the MW scheme implies *non-classical* game even if the players' joint strategy is an unentangled state. In particular, if a player's qubit is in an equal superposition of computational basis states, she cannot affect the game outcome in contrast to her strategic position in the classical game. On the other hand, the players have no impact on the form of the initial state.

We are going to show that the above-mentioned drawbacks vanish by allowing the players to choose between the basis state that corresponds to the classical game and the state $|\Psi\rangle$. In this case of a 2×2 game

$$\begin{pmatrix} (a_{00}, b_{00}) & (a_{01}, b_{01}) \\ (a_{10}, b_{10}) & (a_{11}, b_{11}) \end{pmatrix}, \quad \text{where} \quad (a_{ij}, b_{ij}) \in \mathbb{R}^2. \tag{1}$$

our refinement of the MW scheme for this game is defined on an inner product space $(\mathbb{C}^2)^{\otimes 4}$ by the following components:

- A positive operator H,

$$H = (\mathbb{1}\otimes\mathbb{1} - |11\rangle\langle 11|)\otimes|00\rangle\langle 00| + |11\rangle\langle 11|\otimes|\Psi\rangle\langle\Psi|, \tag{2}$$

  where $|\Psi\rangle \in \mathbb{C}^2\otimes\mathbb{C}^2$ such that $\||\Psi\rangle\| = 1$,

- Players' pure strategies: $P^{(1)}_i\otimes U^{(3)}_j$ for player 1, $P^{(2)}_k\otimes U^{(4)}_l$ for player 2, where $i,j,k,l = 0,1$, and the upper indices identify the subspace $\mathbb{C}^2$ of $(\mathbb{C}^2)^{\otimes 4}$ on which the operators

$$P_0 = |0\rangle\langle 0|,\ P_1 = |1\rangle\langle 1|, \quad U_0 = \mathbb{1},\ U_1 = \sigma_x, \tag{3}$$

  are defined. That is, player 1 acts on the first and third qubit, player 2 acts on the second and fourth one. The order of qubits is in line with the upper indices.

- Measurement operators M_1 and M_2 given by formula

$$M_{1(2)} = \mathbb{1}\otimes\mathbb{1}\otimes\left(\sum_{x,y=0,1} a_{xy}(b_{xy})|xy\rangle\langle xy|\right), \tag{4}$$

  where a_{xy}, b_{xy} are the payoffs from (1).

As a result, we show that the players' strategies in the quantum game do not have to be unitary operators or even superoperators. They may include projectors that determine the state on which the unitary operations are performed. Moreover, the initial state does not have to be a density operator. Certainly, the scheme is in accordance with the laws of quantum mechanics. The resulting state is given by a density operator, and therefore the payoff measurement is well-defined. A positive point of the scheme is a way it can be considered. Given a bimatrix game the scheme outputs a bimatrix game. Consequently, it implies similar complexity in finding optimal strategies for the players.

* P.Frackiewicz@impan.gov.pl

[1] Eisert J Wilkens M and Lewenstein M, Quantum Games and Quantum Strategies *Phys. Rev. Lett.* **83** 3077-80 (1999)
[2] Frackiewicz P, Application of the Eisert-Wilkens-Lewenstein quantum game scheme to decision problems with imperfect recall *J. Phys. A: Math. Theor.* **44** 325304 (2011)
[3] Sharif P and Heydari H, Quantum solution to a three player Kolkata restaurant problem using entangled qutrits *Quantum Inf. Comput.* **14** 295-305 (2014)
[4] Mathur M and Sen D, Coherent states for SU(3) *J. Math. Phys.* **42** 4181-96 (2001)
[5] Marinatto L and Weber T, A quantum approach to static games of complete information *Phys. Lett.* A **272** 291 (2000)