Comparative study of nonclassicality, entanglement, and dimensionality of multimode noisy twin beams

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Nonclassicality (quantumness), quantum entanglement, as well as dimensionality of a multimode noisy twin beam are determined using the characteristic function of the beam written in the Fock basis. One-to-one correspondence between the negativity N, quantifying entanglement, and the nonclassicality depth τ is revealed through the relation:

$$N = \frac{\tau}{1-2\tau}, \tag{1}$$

which is valid even when noise is included.

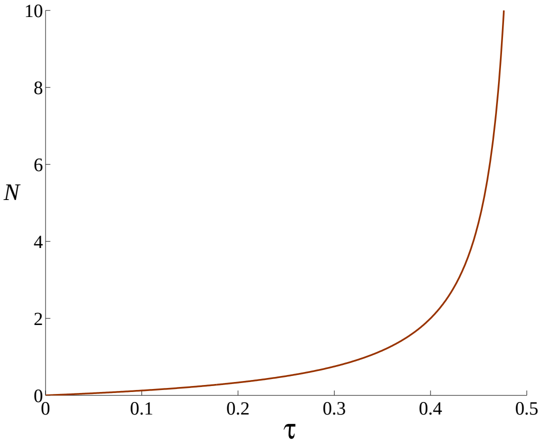

FIG. 1. Negativity N as a function of the nonclassical depth τ, according to Eq. 1.

Twin beams, which are either entangled or nonclassical (independent of their entanglement), are observed only for the limited degrees of noise that degrades their quantumness. This allowed noise, expressed by the mean noise photon numbers in the signal (B_s) or idler (B_i) beams, must obey the inequalities:

$$B_{\rm s} + B_{\rm i} < 1 \quad \text{and} \quad B_{\rm p} > \frac{B_{\rm s} B_{\rm i}}{1-(B_{\rm s}+B_{\rm i})}, \tag{2}$$

where $B_{\rm p}$ is the mean paired photon number.

The dimensionality of the twin beam is quantified by the participation ratio $R_{s(i)}$ for the signal (idler) beam which in turn coincides with the definition of the Schmidt number for pure states [1] and by the dimensionality of entanglement $K_{\rm ent}$ [2]. The comparison between those dimensionality measures is also shown. We introduce the coefficient $r_{\rm ent}$ as the measure of the relative contribution of the entanglement and noise degrees of freedom in the bipartite system:

$$r_{\rm ent} = \frac{2\tilde{K}_{\rm ent}}{R_{\rm s} + R_{\rm i}}, \tag{3}$$

where $\tilde{K}_{\rm ent}$ is an adjusted entanglement dimensionality [3].

The values of coefficient $r_{\rm ent}$ decrease with the increasing values of the mean photon-pair number $B_{\rm p}$. This behavior, however, naturally originates in the fragility of entanglement with respect to the noise. More intense twin beams (with greater values of $B_{\rm p}$) are less resistant to a given amount of the noise compared to low-intensity twin beams. This is explained by larger dimensions of the effectively-populated Hilbert spaces of more intense twin beams, and so more complex structures of the entanglement. As a consequence, relatively higher numbers of the degrees of freedom, serving for describing entanglement in more intense noiseless twin beams are transfered to the parts of the twin beam describing the noise.

To describe the multimode beams, we consider two different cases; namely, the twin beams composed of M independent identical single-mode twin beams and the experimental multimode twin beams. The latter have a more complex structure compared with those in the first case. The reason is that the spatiotemporal modes of twin beams are shared by the signal and idler fields, and so they can be broken before or during the detection owing to the spectral and/or spatial filtering [4]. We also utilize the concept of weak nonclassicality [5], which is also useful for the experimental multimode twin beams considered as composed of one effective paired (macro)mode.

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