# Gaussianity-dependent separability criterion for continuous-variable systems

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Quantum entanglement is nowadays considered a central resource in the field of quantum information and computation [1]. As a consequence, it is crucial to be able to determine whether a state is entangled or separable, which is provably a hard decision problem. In the context of continuous-variable systems, such as bosonic modes or collective atomic spins (in the limit of large ensembles), the separability problem has been addressed by Duan et al. [2] and Simon [3]. Specifically, a necessary criterion for the separability of any two-mode states has been derived, while a necessary and sufficient criterion for separability has been found in the specific case of Gaussian states.

In this work, we investigate an improvement of the Duan-Simon criterion that enables a stronger detection of entanglement for non-Gaussian two-mode states. The improved condition works by imposing an additional constraint on the degree of Gaussianity of the state, defined as [4]

$$g = \frac{\mathrm{Tr}(\rho\rho^G)}{\mathrm{Tr}(\rho^G\rho^G)} \tag{1}$$

where $\rho^G$ is the Gaussian state which has the same covariance matrix as $\rho$ (we restrict to zero-mean states with no loss of generality).

In the case considered here, the Duan-Simon criterion expresses the fact that the EPR-like variance

$$\Delta_{EPR} = \frac{1}{2}\left(\langle(\hat{x}_1+\hat{x}_2)^2\rangle + \langle(\hat{p}_1-\hat{p}_2)^2\rangle\right) \tag{2}$$

must satisfy $\Delta_{EPR} \geq 1$ for any separable state, which yields therefore a necessary condition for separability. Conversely, $\Delta_{EPR} < 1$ is a sufficient condition for entanglement. We apply this condition to a family of (not-necessarily Gaussian) two-modes states, which have a covariance matrix defined by parameters $a$, $\lambda$, and $\eta$ :

$$\gamma = \begin{pmatrix} \left(\frac{a+\lambda^2}{1-\lambda^2}+\eta\right)\mathbb{1} & \frac{(a+1)\lambda}{1-\lambda^2}\,\sigma_z \\ \frac{(a+1)\lambda}{1-\lambda^2}\,\sigma_z & \frac{a\lambda^2+1}{1-\lambda^2}\,\mathbb{1} \end{pmatrix}. \tag{3}$$

(The Gaussian state characterized by $\gamma$ can be obtained by injecting the vacuum state and a thermal state with covariance matrix $a\mathbb{1}$ in a two-mode squeezer of parameter $\lambda$, and then adding a Gaussian noise of variance $\eta$ on the first mode.) The Duan-Simon procedure consists in applying a partial transposition on the state, which means applying the map $p_2 \to -p_2$, and then checking that one reaches a physical state; otherwise, the initial state is declared entangled. The physicality of the partially-transposed state is checked by verifying that its two symplectic eigenvalues satisfy the conditions $\nu_{1,2} \geq 1$.

The key observation here is that the latter inequalities boil down to expressing the uncertainty principle on both modes, namely $\det(\gamma_{1,2}) = (\nu_{1,2})^2 \geq 1$. However, it has been shown in [4] that a tighter lower bound may be obtained if $g$ is known (it is equal to one for $g = 1$, but larger than one for non-Gaussian states). Applying this result on the two modes of the partially-transposed state, we obtain a stronger test of its physicality, which depends on the degree of Gaussianity of the state. This leads the set of states that sit at the edge of separability for a specific degree of Gaussianity, as illustrated in Figure 1.

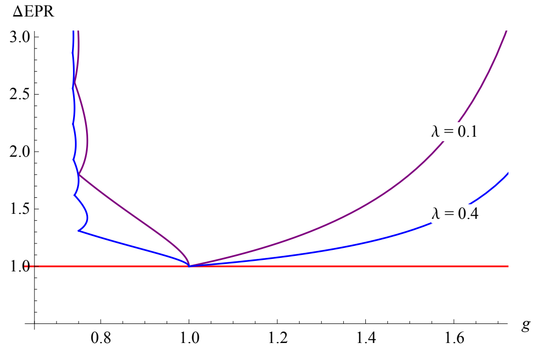


FIGURE 1. Necessary separability criterion for non-Gaussian states. Plots of the $\Delta_{EPR}$ threshold value as a function of the degree of Gaussianity $g$, for different values of the squeezing parameter $\lambda$. The red line represents Duan-Simon criterion.

If a state is associated with a value of $\Delta_{EPR}$ below the plotted curve, it is an entangled state. The Duan-Simon criterion corresponds to a constant line at $\Delta_{EPR} = 1$. Thus, for non-Gaussian states (with $g \neq 1$), we have found a stronger separability criterion, allowing us to detect more entangled states.

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