Regular phase operator and a new number-phase Weyl-Wigner correspondence for the photon

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Recently we have introduced a new 'exponential phase operator' F, defined by the 'polar decomposition' $A = F\sqrt{N+\nu}$ of the quantized amplitude of a linear oscillator, which may also represent a mode of the radiation field [1]. Here ν is a positive parameter, which is connected with the Bargmann index of the SU(1,1) coherent states associated to F. By the Fourier correspondence $e^{i\varphi} \to F$ we have constructed a regular phase operator Φ as a strongly convergent series, which is an analogon of the quantum saw-tooth phase operator Φ_{GW}, originally introduced by Garrison and Wong [2]. We have also derived the generalied spectral resolution of the phase operator Φ, on the basis of which new phase probability distribution function and phase probability density function have been introduced.

In the present work we extended this formalism, and introduced a new number-phase Weyl-Wigner correspondence between operators and phase-space functions. We shall show in several examples that the new Wigner functions and Weyl-Wigner transforms behave 'regularly' in comparison with the so-called special Weyl-Wigner associates introduced earlier by Vaccaro [3]. As an illustration, in Figures 1 and 2 below, we compare the number-phase Weyl transform of the Garrison-Wong operator and of that of the regular phase operator, introduced by us [1].

The present formalism may be applied in quantum signal analysis, or may for example be useful in describing the quantum phase properties of extreme radiation fields like attosecond pulses.

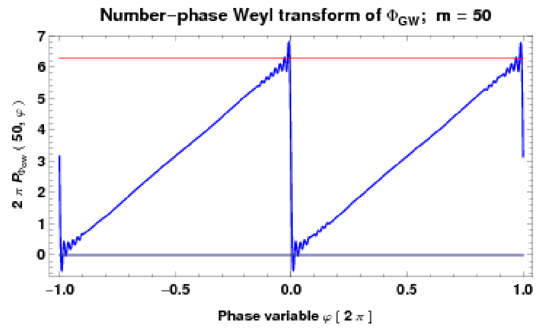

FIG. 1. Shows the number-phase Weyl transform $2\pi P_{\Phi_{GW}}(m,\varphi)$ of the Garrison and Wong phase operator Φ_{GW} as a function of φ (measured in 2π radian units), for the photon number value $m = 50$. We have shown that - according to the formalism due to Vaccaro [3] this is, in fact, the $m-th$ partial sum s_{50} of the famous Fourier series of the classical saw-tooth function $\Phi_{cl}(e^{i\varphi}) = \pi - 2\sum_{k=l}^{\infty}(sin(k\varphi))/k = \varphi$. Horizontal lines are also drawn for reference, at the minimum and maximum values, $\Phi_{cl} = 0$ and $\Phi_{cl} = 2\pi$, respectively. The Gibbs phenomenon at the discontinuity points $\varphi = -2\pi, 0, 2\pi$ are clearly seen.

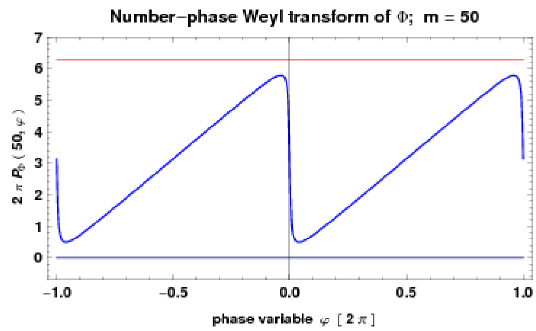

FIG. 2. Shows the number-phase Weyl transform $2\pi P_{\Phi}(m,\varphi)$ of the regular phase operator Φ [1] as a function of φ (measured in 2π radian units), for the photon number value $m = 50$. In contrast to the Weyl transform of the Garrison-Wong phase operator shown in Fig. 1, this function does not suffer of the Gibbs phenomenon, and behaves regularly; it stays within the upper an lower bounds and converges uniformly to φ.

Acknowledgments

The author thanks many interesting and fruitful discussions with Prof. Dr. Wolfgang Schleich on the quantum phase problem. This work has been supported by the Hungarian Scientific Research Foundation OTKA, Grant No. K 104260. Partial support by the ELI-ALPS project is also acknowledged. The ELI-ALPS project (GOP-1.1.1-12/B-2012-0001) is supported by the European Union and co-financed by the European Regional Development Fund.

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