Efficient noiseless amplification for light fields with larger amplitudes

Jinwoo Park,[1] Jaewoo Joo,[2] Alessandro Zavatta,[3,4] Marco Bellini,[3,4] and Hyunseok Jeong[1,*]

[1]*Center for Macroscopic Quantum Control, Department of Physics and Astronomy, Seoul National University, Seoul 151-742, Republic of Korea*
[2]*Advanced Technology Institute and Department of Physics, University of Surrey, Guildford, GU2 7XH, United Kingdom*
[3]*Istituto Nazionale di Ottica, INO-CNR, Largo Enrico Fermi, 6, I-50125 Firenze, Italy*
[4]*Department of Physics and LENS, University of Firenze, 50019 Sesto Fiorentino, Firenze, Italy*

We propose and investigate a scheme for non-deterministic noiseless linear amplification [1–4] of coherent states using two-photon addition. The noiseless amplifications considered here are $\hat{a}\hat{a}^\dagger$ and $\hat{a}^{\dagger 2}$, where $\hat{a}$ ($\hat{a}^\dagger$) is the photon annihilation (addition) operator. The $\hat{a}\hat{a}^\dagger$ operation as the first order approximation of the ideal noiseless amplification works well for small-amplitude coherent states [2, 4]. It was pointed out that n-photon addition, $(a^\dagger)^n$, may be used to amplify coherent states [5]. Here we find that the $a^{\dagger 2}$ operation is an effective noiseless amplifier for coherent states with relatively larger amplitudes. Figures of merit examined here are the state fidelity, the amplitude gain, and the equivalent input noise (EIN) [6]. The noiseless property of the amplification is assessed by the EIN of the amplifier, which is affected by both the state fidelity and the amplitude gain.

The ($\hat{a}\hat{a}^\dagger$)-amplification always exhibits higher maximum fidelity than the $(\hat{a}^\dagger)^2$-amplification while it is the opposite for the amplitude gain. The maximum fidelity between the $\hat{A}$-amplified ($\hat{A} \in \{\hat{a}\hat{a}^\dagger, \hat{a}^{\dagger 2}\}$) coherent state of initial amplitude $|\alpha_i|$ and the coherent state of the final amplitude $|\alpha_f|$ is numerically calculated and presented in Fig. 1(a). The amplitude gain from the amplification $\hat{A}$ can be defined as the input and output ratio of the expectation values of the quadrature operator with phase λ, $\hat{x}_\lambda$ [4]: $g_\lambda^{\hat{A}} = \langle x_\lambda \rangle_{\text{out}} / \langle x_\lambda \rangle_{\text{in}}$. The amplitude gain is independent of λ for the amplifications of coherent states considered here. The gain monotonically decreases to unity with respect to $|\alpha_i|$ as shown in Fig. 1(b).

We employ the EIN [6] for comparison between the two amplification schemes. The EIN of an amplifier is defined as

$$E_\lambda^{\hat{A}} = \frac{\langle \Delta x_\lambda \rangle_{\text{out}}^2}{(g_\lambda^{\hat{A}})^2} - \langle \Delta x_\lambda \rangle_{\text{in}}^2, \qquad (1)$$

where $\langle \Delta x_\lambda \rangle^2$ is the variance of the $\hat{x}_\lambda$. It represents the level of noise added into the input signal to mimic the output signal in the quadrature x_λ. The λ-averaged EINs, $E^{\hat{a}\hat{a}^\dagger}(\alpha_i)$ and $E^{a^{\dagger 2}}(\alpha_i)$, are numerically computed and plotted in Fig. 1(c). All average EINs are negative, which indicates the characteristic of noiseless amplification; negative EIN cannot be obtained by the classical amplification. As the $\hat{a}\hat{a}^\dagger$-amplification has much higher fidelity than $\hat{a}^{\dagger 2}$ for small $|\alpha_i|$, $\hat{a}\hat{a}^\dagger$ has lower EIN for small $|\alpha_i|$, while $\hat{a}^{\dagger 2}$ has lower EIN for large $|\alpha_i|$ due to higher gains.

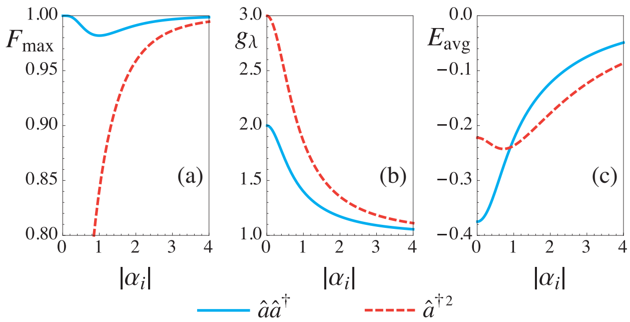

FIG. 1. (a) Maximum fidelities, (b) amplitude gains, and (c) average EINs when the amplification $\hat{a}\hat{a}^\dagger$ (solid curve) and $\hat{a}^{\dagger 2}$ (dashed curve) are applied to the coherent state of initial amplitude $|\alpha_i|$.

The $\hat{a}^{\dagger 2}$-amplifications also serve as an effective noiseless amplification for superpositions of coherent states (SCSs), where similar behaviors in the state fidelities, amplitude gains, and EINs can be observed. The enhancement in the maximum fidelity can be found when a squeezed state is used to approximate a superposition of coherent states for the amplifications considered here.

In terms of EIN, $\hat{a}^{\dagger 2}$ is better than $\hat{a}\hat{a}^\dagger$ as a noiseless amplifier for coherent states as far as the initial amplitude is $|\alpha_i| \gtrsim 0.91$. Similar behaviors are found for the SCSs and squeezed states as their approximations. In contrast to the previous studies [1–4], our work provides efficient schemes to implement a noiseless amplifier for light fields with medium and large amplitudes.

* h.jeong37@gmail.com

[1] T. C. Ralph and A. P. Lund, arXiv:0809.0326.
[2] J. Fiurášek, Phys. Rev. A **80**, 053822 (2009).
[3] G. Y. Xiang, T. C. Ralph, A. P. Lund, N. Walk and G. J. Pryde, Nature Photonics **4**, 316 (2010).
[4] A. Zavatta, J. Fiurášek, and M. Bellini, Nature Photon. **5**, 52 (2011).
[5] H. Jeong, A. Zavatta, M. Kang, S.-W. Lee, L. S. Costanzo, S. Grandi, T. C. Ralph, and M. Bellini, Nature Photon. **8**, 564 (2014).
[6] P. Grangier, J.-M. Courty, and S. Reynaud, Optics Communications **89**, 99 (1992).