Measuring Quantum Coherence with Entanglement

Alexander Streltsov,[1] Uttam Singh,[2] Himadri Shekhar Dhar,[2,3] Manabendra Nath Bera,[2] and Gerardo Adesso[4,*]
[1]*ICFO-The Institute of Photonic Sciences, Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain*
[2]*Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211 019, India*
[3]*School of Physical Sciences, Jawaharlal Nehru University, New Delhi 110 067, India*
[4]*School of Mathematical Sciences, The University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom*

Coherence is a fundamental aspect of quantum physics that encapsulates the defining features of the theory, from the superposition principle to quantum correlations. It is a key component in various quantum information and estimation protocols and is primarily accountable for the advantage offered by quantum tasks versus classical ones. In general, coherence is an important physical resource in low-temperature thermodynamics, for exciton and electron transport in biomolecular networks, and for applications in nanoscale physics. Experimental detection of coherence in living complexes and creation of coherence in hot systems have also been reported.

While the theory of quantum coherence is historically well developed in quantum optics in terms of quasiprobability distributions and higher-order correlation functions, a rigorous framework to quantify coherence for general states adopting the language of quantum information theory has only been attempted in recent years. This framework is based on the characterization of the set of incoherent states and a class of 'free' operations, named incoherent quantum channels, that map the set onto itself [1]. The resulting resource theory of quantum coherence is in direct analogy with the resource theory of quantum entanglement, in which local operations and classical communication are identified as the 'free' operations that map the set of separable states onto itself. Within such a framework for coherence, one can define suitable measures that vanish for any incoherent state, and satisfy specific monotonicity requirements under incoherent quantum channels. Measures that respect these conditions gain the attribute of coherence monotones, in analogy with entanglement monotones.

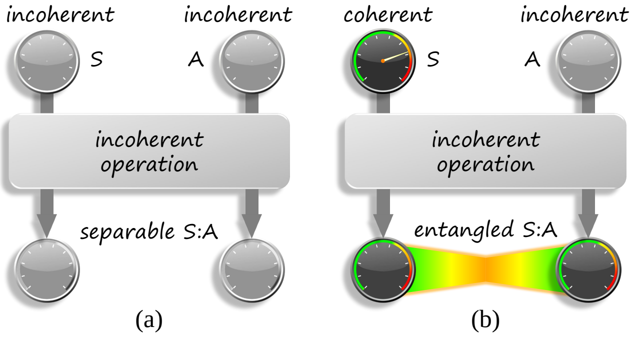

FIG. 1. **Equivalence between coherence and entanglement.** (a) Incoherent operations cannot create entanglement from incoherent input states. (b) Entanglement can instead be created by incoherent operations if at least one of the inputs is coherent. We show that all coherent input states of a system S are useful for entanglement creation via incoherent operations on S and an incoherent ancilla A. Input coherence and output entanglement are quantitatively equivalent: For every entanglement monotone E, the maximum entanglement that can be created between S and A by incoherent operations defines a faithful measure of coherence C_E in the initial state of S.

Both coherence and entanglement intuitively capture the quantumness of a physical system, and it is well known that entanglement stems from the superposition principle, which is also the essence of coherence. It is then legitimate to ask how can one resource emerge *quantitatively* from the other. In this work [2], we provide a mathematically rigorous approach to resolve the above question. In particular, in our central result we show that any nonzero amount of coherence in a system S can be 'activated' into (distillable) entanglement between S and an initially incoherent ancilla A, by means of incoherent operations (see Fig. 1). This establishes coherence as a universal resource for entanglement creation. In quantitative terms, given a distance-based pair of quantifiers for coherence and entanglement, we show that the initial degree of coherence of S bounds from above the entanglement that can be created between S and A by any incoherent operation. Conversely, our scheme also reveals a novel, general quantification of coherence in terms of entanglement creation. Namely we prove that, given an arbitrary set of entanglement monotones $\{E\}$, one can define a corresponding class of coherence monotones $\{C_E\}$ that satisfy all the requirements of Ref. [1]. The input coherence C_E of S is specifically defined as the maximum output entanglement E over all incoherent operations on S and A. Altogether, these results demonstrate a fundamental qualitative and quantitative *equivalence* between coherence and entanglement, and provide an intuitive operational scheme to interchange these two nonclassical resources for suitable applications in quantum technologies.

---

[*] gerardo.adesso@nottingham.ac.uk

[1] T. Baumgratz, M. Cramer, and M. B. Plenio, Phys. Rev. Lett. **113**, 140401 (2014).
[2] A. Streltsov, U. Singh, H. S. Dhar, M. N. Bera, and G. Adesso, arXiv:1502.05876 (2015).