Controlling entropic uncertainty bounds through memory effects

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One of the defining traits of quantum mechanics is the uncertainty principle which puts a bound on our ability to predict the measurement results of two incompatible observables simultaneously. In its original form, the uncertainty relation has been expressed in terms of the standard deviations of two observables Y and Z [1, 2]

$$\Delta Y \Delta Z \geq \frac{1}{2} |\langle\psi|[Y,Z]|\psi\rangle|. \tag{1}$$

An alternative method to quantify uncertainty is based on entropic measures instead of standard deviations. Such an approach is especially meaningful when we are interested in the uncertainty related to the lack of knowledge of possible measurement outcomes. One of the most well known entropic uncertainty relations [3] can be written as

$$H(Q) + H(R) \geq \log_2 \frac{1}{c}, \tag{2}$$

where $H(X) = -\sum_x p(x) \log_2 p(x)$ quantifies the amount of uncertainty about the observable X before the result of its measurement is known to us. Here, the probability of the outcome x is denoted by $p(x)$ when a state ρ is measured in X-basis. Complementarity of the observables Q and R is given by $1/c = 1/\max_{i,j} |\langle\psi_i|\phi_j\rangle|^2$, where $|\psi_i\rangle$ and $|\phi_j\rangle$ are the eigenstates of the observables Q and R, respectively.

If we allow Bob to have access to an additional particle serving as a quantum memory (particle B), which is entangled with the particle held by Alice (particle A) to be measured by incompatible observables Q and R, then it is proved [4] that a stronger uncertainty relation holds

$$S(Q|B) + S(R|B) \geq \log_2 \frac{1}{c} + S(A|B), \tag{3}$$

where $S(\rho) = \mathrm{tr}[\rho \log_2 \rho]$ is the von Neumann entropy. While $S(Q|B)$ and $S(R|B)$ denote the conditional von Neumann entropies of the post-measurement states after the subsystem A is measured by Q and R respectively, $S(A|B)$ is the conditional entropy of the composite system AB. In particular, the left-hand side of Eq. (3) quantifies the total amount of ignorance as measured in terms of entropy about the observables Q and R. On the other hand, there appears an extra term on the right-hand side, namely $S(A|B)$, modifying the lower bound of the total amount of uncertainty associated to the observables Q and R.

In our work, we consider a more realistic physical setting where the memory particle B is an open system, rather than being an isolated system, interacting with an environment E and thus undergoing a decoherence process through a channel Φ_t. Assuming that the state of the composite system AB is initially pure, and exploiting the relation of coherent information $I_c(\rho_B, \Phi_t)$ to the conditional von Neumann entropy $S(A|B)$, we provide a direct link between non-Markovian memory effects, defined in terms of the non-monotonical behavior of the quantum mutual information $I(\rho_{AB})$ under local CPTP maps [5], and the rate of change of conditional von Neumann entropy $S(A|B)$. Considering that complementarity $1/c$ has no dependence on the dynamics of the composite system AB, we reveal that memory effects directly control the lower bound of uncertainty of the observables Q and R. Our approach establishes a general connection between the memory effects emerging due to the interaction with environment E and the lower bound of the entropic uncertainty relation, independently of the specific type of non-Markovian noise on the memory particle. We demonstrate the implications of our findings for non-Markovian dephasing and relaxation channels. Moreover, we discuss the role of the flow of information among the constituents of the tripartite system ABE in the link between memory effects and the uncertainty relation [6].

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[1] W. Heisenberg, Z. Phys. **43**, 173 (1927).
[2] H.P. Robertson Phys. Rev. **34**, 163 (1929).
[3] H. Maasen and J.B.M. Uffink, Phys. Rev. Lett. **60**, 1103 (1988).
[4] M. Berta et al., Nat. Phys. **6**, 659 (2010).
[5] S. Luo, S. Fu, and H. Song, Phys. Rev. A **86**, 044101 (2012).
[6] S. Haseli et al., Phys. Rev. A. **90**, 052118 (2014).