

Percolation assisted excitation transport in discrete-time quantum walks on a ring

Martin Štefaňák,^{1,*} Jaroslav Novotný,¹ and Igor Jex¹

¹*Department of Physics, Faculty of Nuclear Sciences and Physical Engineering,
Czech Technical University in Prague, Břehová 7, 115 19 Praha 1 - Staré Město, Czech Republic*

Quantum walks (see e.g. [1] for a recent review) emerged as an extension of a concept of a random walk to a unitary evolution of a quantum particle on a discrete graph or lattice. They have found promising applications in quantum information processing [2, 3] and models of coherent transport on networks [4, 5]. Our present paper contributes to the latter area of research.

We analyze a simple model of excitation transport on a ring with vertices labeled from $-N$ to $N+1$. The excitation enters the ring on the vertex 0 and hops to the neighbouring vertices in discrete time steps. The propagation through the ring is modeled by a coined quantum walk. Opposite of the starting vertex, i.e. on the vertex $N+1$, is the sink which carries the excitation away from the ring. We study the asymptotic transport efficiency to the sink η defined by

$$\eta = 1 - \lim_{t \rightarrow \infty} P_T(t), \quad (1)$$

where $P_T(t)$ is the probability that the excitation remains on the ring at time t . We show that if the propagation is modeled by a two-state walk, i.e. the excitation has to jump from its present position to the neighbouring sites, the transport efficiency η is unity. Moreover, the probability to remain at the ring $P_T(t)$ decays exponentially in the number of steps t with the decay constant depending on the coin operator.

However, if the excitation is, in addition, allowed to stay at its present position, i.e. the propagation is modeled by a three-state quantum walk, then part of the wave-packet can be trapped in the vicinity of the origin and never reaches the sink. This phenomenon called trapping (or localization) was first observed for the three-state Grover walk on infinite line [6, 7] and recently analyzed for a broader variety of coin operators in [8–11]. The source of trapping is two-fold. First, the evolution operators of certain three-state quantum walks on infinite line have point spectrum, i.e. a flat-band in the momentum picture. Second, the corresponding eigenstates are spatially localized. Hence, this feature is found also for quantum walks on finite rings, even in the presence of the sink. The consequence of the trapping effect is that the excitation transport is not efficient, i.e. $\eta < 1$, since the probability to remain at the ring $P_T(t)$ has a non-vanishing limit. Nevertheless, we show that for some three-state quantum walks the trapping can be eliminated by dynamical percolation of the ring [12], i.e. when we allow the edges between the vertices to break randomly at each time step. Hence, by dynamical percolation of the ring efficient excitation transport can be achieved.

To be more specific, we focus on the quantum walks driven by a two-parameter set of coins

$$C = \begin{pmatrix} -\rho^2 & \rho\sqrt{2-2\rho^2} & e^{-i\gamma}(1-\rho^2) \\ \rho\sqrt{2-2\rho^2} & 2\rho^2-1 & e^{-i\gamma}\rho\sqrt{2-2\rho^2} \\ e^{i\gamma}(1-\rho^2) & e^{i\gamma}\rho\sqrt{2-2\rho^2} & -\rho^2 \end{pmatrix}, \quad (2)$$

which show the trapping effect [8]. The parameter ρ determines the speed of propagation of the excitation through the ring. In the case of unpercolated walk, the second parameter γ does not play a role since it can be transformed away by a suitable choice of the basis in the coin space [10]. However, it crucially affects the percolated walk. We show that for $\gamma = 0$, the eigenvectors corresponding to the flat-band are attractors of the percolated walk [13, 14]. In such a case, trapping is stable under percolation and the transport efficiency is not improved. On the other hand, for $\gamma \neq 0$ the attractor conditions [13, 14] are not fulfilled. Hence, the dynamical percolation of the ring eliminates the trapping effect and efficient excitation transport is achieved.

* martin.stefanak@fjfi.cvut.cz

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