Pairing in a system of a few ultra-cold attractive fermions in a harmonic trap

Tomasz Sowiński,[1,2,*] Mariusz Gajda,[1,2] and Kazimierz Rzążewski[2]

[1] *Institute of Physics of the Polish Academy of Sciences Al. Lotników 32/46, 02-668 Warsaw, Poland*

[2] *Center for Theoretical Physics of the Polish Academy of Sciences Al. Lotników 32/46, 02-668 Warsaw, Poland*

We study a strongly attractive system of a few spin-(1/2) fermions confined in a one-dimensional harmonic trap, interacting via two-body contact potential. The Hamiltonian of the system studied reads:

$$\hat{\mathcal{H}} = \sum_{\sigma\in\{\downarrow,\uparrow\}} \int \mathrm{d}x\, \hat{\Psi}^\dagger_\sigma(x) \left[-\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{2}x^2\right] \hat{\Psi}_\sigma(x) + g\int \mathrm{d}x\, \hat{\Psi}^\dagger_\downarrow(x)\hat{\Psi}^\dagger_\uparrow(x)\hat{\Psi}_\uparrow(x)\hat{\Psi}_\downarrow(x), \quad (1)$$

where $\Psi_\sigma(x)$ annihilates a fermion with spin σ at a point x, and g measures a strength of the contact interactions. Performing exact diagonalization of the Hamiltonian we analyze the ground state $|\mathtt{G}_0\rangle$ and the thermal state $\hat{\rho}_\mathrm{T} = \sum_i p_i |\mathtt{G}_i\rangle\langle\mathtt{G}_i|$ (with $p_i = \exp(E_i/T)/\sum_i \exp(E_i/T)$) of the system in terms of one- and two-particle reduced density matrices:

$$\rho^{(1)}_\mathrm{T}(x;x') = \frac{2}{N}\times\mathrm{Tr}\left[\hat{\rho}_\mathrm{T}\,\hat{\Psi}^\dagger_\sigma(x)\hat{\Psi}_\sigma(x')\right], \quad (2a)$$

$$\rho^{(2)}_\mathrm{T}(x_1,x_2;x_1',x_2') = \frac{4}{N^2}\times\mathrm{Tr}\left[\hat{\rho}_\mathrm{T}\,\hat{\Psi}^\dagger_\downarrow(x_1)\hat{\Psi}^\dagger_\uparrow(x_2)\hat{\Psi}_\uparrow(x_2')\hat{\Psi}_\downarrow(x_1')\right]. \quad (2b)$$

We show how for strong attraction the correlated pairs emerge in the system. We find that the fraction of correlated pairs depends on temperature and we show that this dependence has universal properties analogous to the gap func- tion known from the theory of superconductivity. In contrast to the standard approach based on the variational ansatz and/or perturbation theory, our predictions are exact and are valid also in a strong-attraction limit. Our findings contribute to the understanding of strongly correlated few-body systems and can be verified in current experiments.

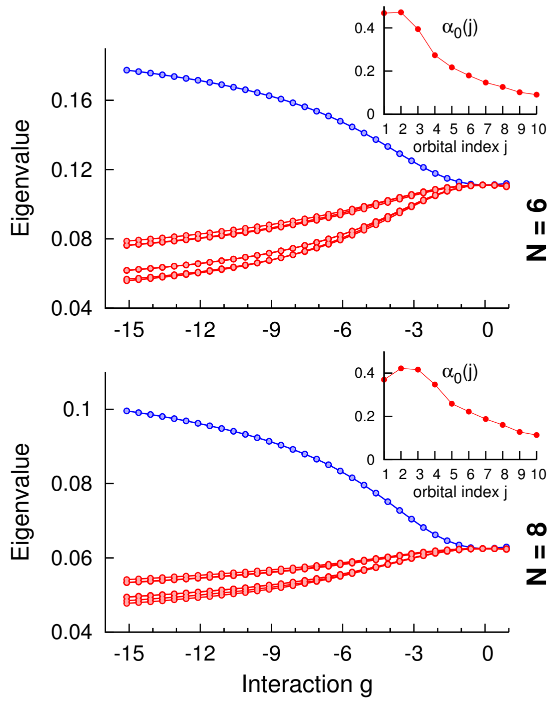

FIG. 1. First eight eigenvalues λ_i of the reduced two-fermion density matrix $\rho^{(2)}_\mathrm{T}$ as a function of interaction g, calculated in the ground-state of the system $|\mathtt{G}_0\rangle$ obtained numerically for $N_\uparrow = N_\downarrow = 3$ and 4. For attractive interactions one of the eigenvalues starts to dominate in the system indicating occurrence of the fermionic pair condensation. In insets we show probability amplitudes $\alpha_0(j)$ for the dominant orbital of the correlated pair for $g = -10$.

* Tomasz.Sowinski@ifpan.edu.pl

[1] T. Sowiński, M. Gajda, and K. Rzążewski, EPL **109**, 26005 (2015).