Optimal environment for quantum bosonic Gaussian channels

Evgueni Karpov,[1,*] Joachim Schäfer,[1] Oleg V. Pilyavets,[1] and Nicolas J. Cerf[1]

[1]*QuIC, Ecole Polytechnique de Bruxelles, CP 165, Université Libre de Bruxelles (ULB), 1050 Brussels, Belgium*

The capacity of information transmission channels is the central problem of information theory since the foundational works of Shannon. In last decades this problem attracted much attention in the framework of quantum information where different capacities of quantum channels can be defined. In particular, recent advances in the study of bosonic Gaussian quantum channels gave meaning to the study of the Gaussian (classical) capacity of bosonic quantum channels with Gaussian noise. This reduces the optimization problem determining the classical capacity of these channels to the set of Gaussian states and Gaussian encodings. In particular, it was shown that if the minimum entropy at the output of Gaussian channels with phase-symmetric noise is attained by the vacuum (coherent) input state then the Gaussian capacity is equivalent to the classical capacity [1] for any Gaussian noise. The Gaussian minimum output entropy conjecture itself was also recently proven [2]. As in classical case the meaningful definition of the classical capacity for the Gaussian channels can be given only under the input energy constraint because otherwise the capacity is infinite. This makes the capacity of a given channel being a function of the input energy constraint so that this constraint is a parameter determining the optimal input state. Among other such parameters are the ones determining the Gaussian noise, which is usually given. Recently the capacity problem was posed in a larger sense [3] by asking also a question about an optimal environment given a constraint on the noise energy. The intuition following the result obtained by Holevo et al. [4] suggests that the optimal channel noise maximizing the capacity must be squeezed. We prove that this is true for the additive Gaussian noise. We show first that one-shot Gaussian capacity of single mode channel is maximized when all noise energy is concentrated in one quadrature. Further we prove that one-shot Gaussian capacity of for two-mode channel is maximized when all noise energy is concentrated in one quadrature of one mode. Finally, the result for arbitrary number of modes and even in the infinite limit follows by induction. The parallel channels with different environment in each mode arise, in particular, in the context of channels with memory, which is represented by correlated noise [3, 5, 6]. We discuss as well the optimal environment for other main classes of Gaussian channels. The inverse problem of the "worst case" environment for a given noise energy constraint is also considered.

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* Evgueni.Karpov@ulb.ac.be

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