State Reconstruction of an Oscillator Network in an Optomechanical Setting

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The scheme developed in this work allows the full quantum state of a network of harmonically interacting mechanical oscillators to be reconstructed. This is accomplished by coupling one distinguished oscillator of the network via radiation pressure to the optical mode of a cavity. This minimal requirement of only one coupled oscillator renders the scheme relatively noninvasive of the mechanical state.

The Hamiltonian for the system, after linearisation of the coupling, may be written:

\[
H = H_0 + H_{\text{int}}
\]

\[
H_0 = \sum_n \omega_n b_n^\dagger b_n + \sum_{n<m} J_{nm} (b_n b_m^\dagger + b_m b_n^\dagger)
\]

\[
+ \sum_{n<m} K_{nm} (b_n b_m + b_m^\dagger b_n^\dagger)
\]

\[
H_{\text{int}} = g(t) X (b_1 + b_1^\dagger)
\]

where \( b_n \) are the mechanical modes, \( \omega_n \) are the mechanical frequencies, \( J_{nm} \) and \( K_{nm} \) are the coupling constants for the network, \( g(t) \) is a time-dependent optomechanical coupling strength, \( X = a + a^\dagger \) is the optical mode's position quadrature and the oscillator labelled 1 is the distinguished one coupled to the optical mode.

Define \( S \) as the symplectic matrix that brings the network into the basis of normal modes:

\[
S = \begin{pmatrix} S_1 & S_2 \\ S_2^\dagger & S_1^\dagger \end{pmatrix}.
\]

The oscillator network may be transformed into the basis of normal modes, revealing

\[
H_0 = \sum_n \nu_n G_n^d d_n
\]

\[
H_{\text{int}} = g(t) X \sum_n G_n d_n + G_n^* d_n^\dagger
\]

where \( G_n = (S_1 - S_2) n_1 \), \( d_n \) are the normal modes of the network and \( \nu_n \) its eigenfrequencies. Considering an interaction picture defined by \( H_0 \), we have

\[
H_I = g(t) X \sum_j h_j(t)
\]

where \( h_j = G_j d_j e^{-i\nu_j t} + G_j^* d_j^\dagger e^{i\nu_j t} \). The dynamics of the system is solved by

\[
U = e^{i\Psi x^2} D(X \beta)
\]

where \( \Psi = \sum_j \psi_j, \beta = (\beta_1, \beta_2, \ldots, \beta_N)^\dagger \) and

\[
\beta_j = -i G_j \int_0^t g(s) e^{i\nu_j s} ds
\]

\[
\psi = -\int_0^t \text{Im} (\beta_j \beta_j^\dagger) ds
\]

with \( N \) the number of oscillators.

Define a measurement operator \( L = -4\Psi X + P \) (with \( P \) the momentum conjugate with \( X \)) and the mechanical mode quadratures by \( Q_{\beta_j} = d_j e^{-i\theta_j} + d_j^\dagger e^{i\theta_j} \) [where \( \theta_j = \text{arg}(\beta_j) + \frac{\pi}{2} \)]. Note that these quadratures are defined in terms of the normal mode operators. Given an initial separable state \( \rho = |0\rangle \langle 0| \otimes \rho_0 \) with \( |0\rangle \) the optical vacuum and \( \rho_0 \) an arbitrary state of the network, after an appropriate evolution a measurement of the light is made. A histogram of such measurement results produces a set of statistical moments which are related to the moments of the quadratures \( Q_{\beta_j} \) via the following relation:

\[
\langle L^n \rangle = \sum_{k_0+k_1+\ldots+k_N=n} \left( \begin{array}{c} n \\ k_0, k_1, \ldots, k_N \end{array} \right) |0\rangle \langle 0| P^{k_0} \langle 0| \]

\[
(11 \leq j \leq N(-2|\beta_j|Q_{\beta_j})^k \rangle
\]

Inverting the relation above it is possible to obtain a suitable collection of moments for a particular quadrature. The latter can be selected by changing the time profile of the interaction strength \( g(t) \). Thus a suitable set of marginals can be extracted from which, in turn, the phase space distribution of the network can be reconstructed using standard tomographic techniques.